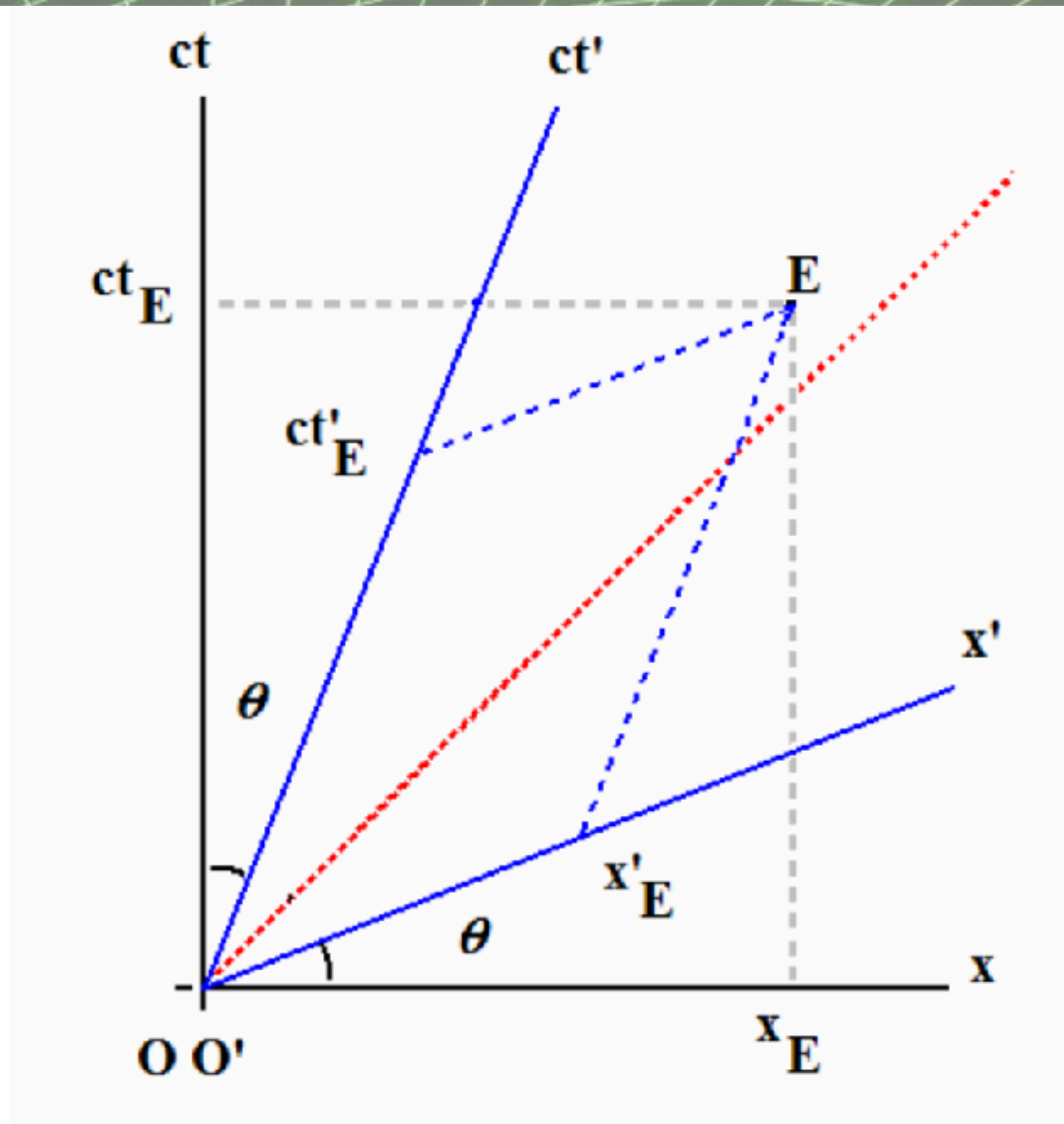


Relativity

Saturday Morning Physics

Patrick Fox





Please, please,
please ask
questions

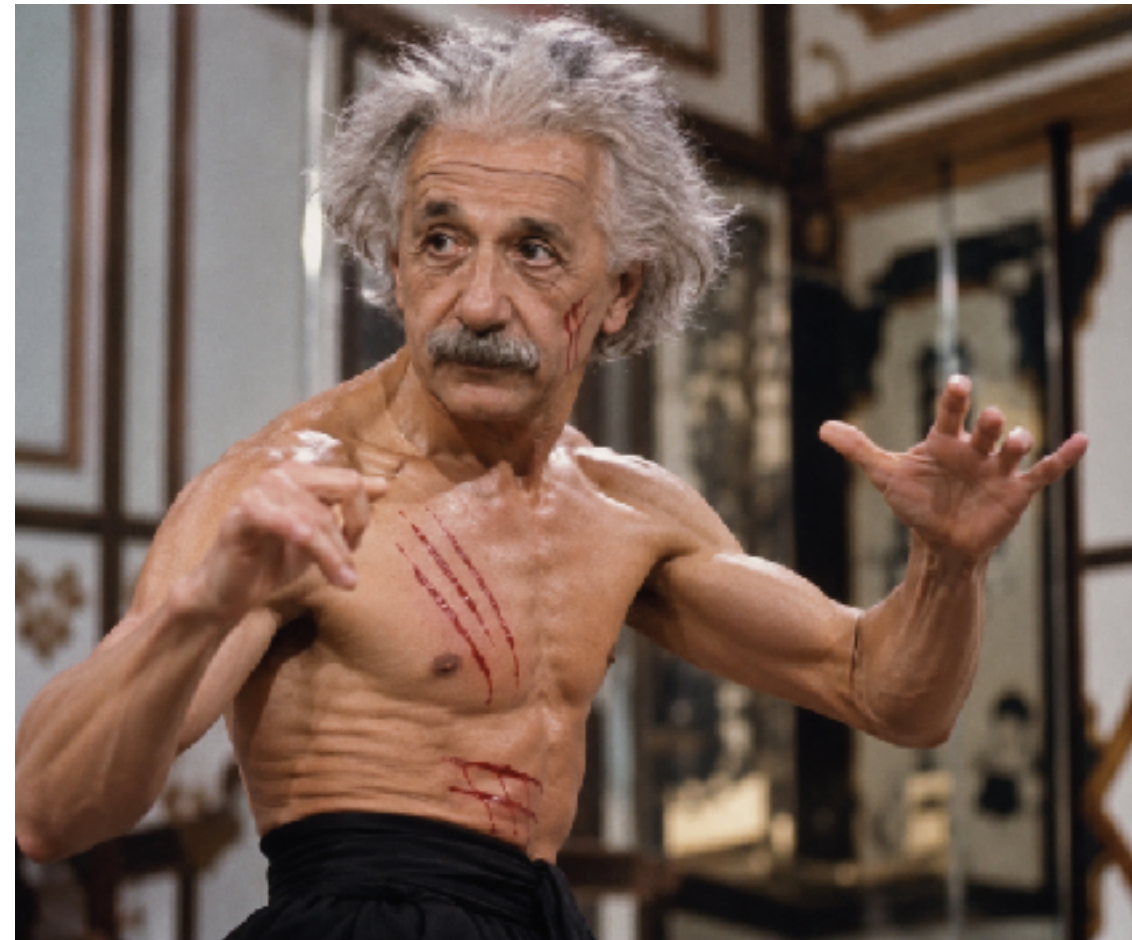


Contents

- Some history
- Failures of classical physics
- Enter Einstein
- Special Relativity
- General Relativity

Contents

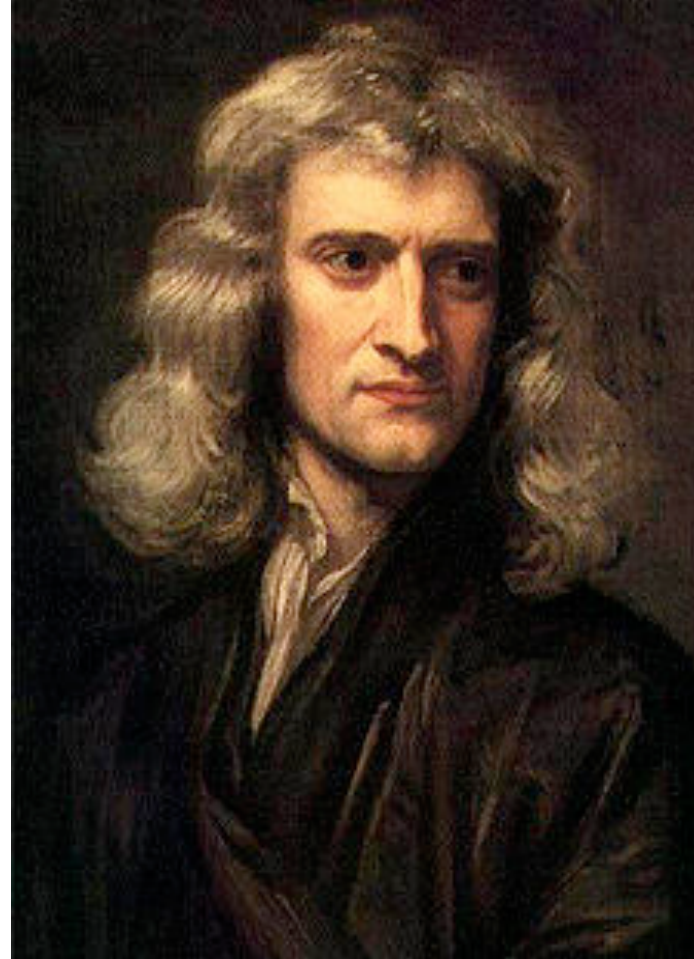
- Some history
- Failures of classical physics
- Enter Einstein
- Special Relativity
- General Relativity



Classical Physics



Galileo

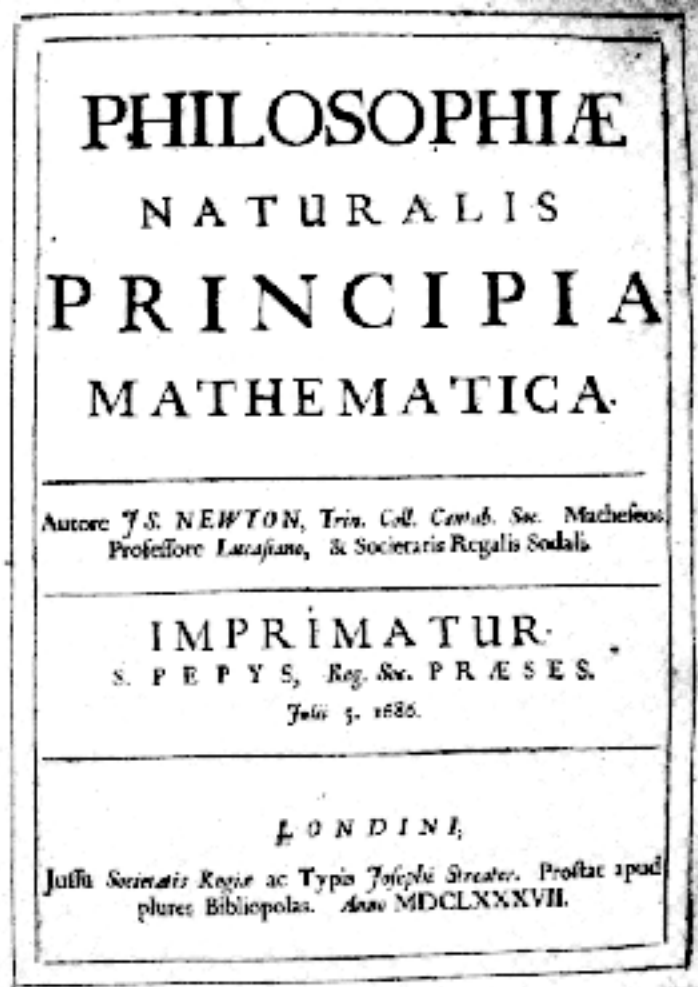


Newton



Maxwell

(+ friends) explains all of physics up to ~1900

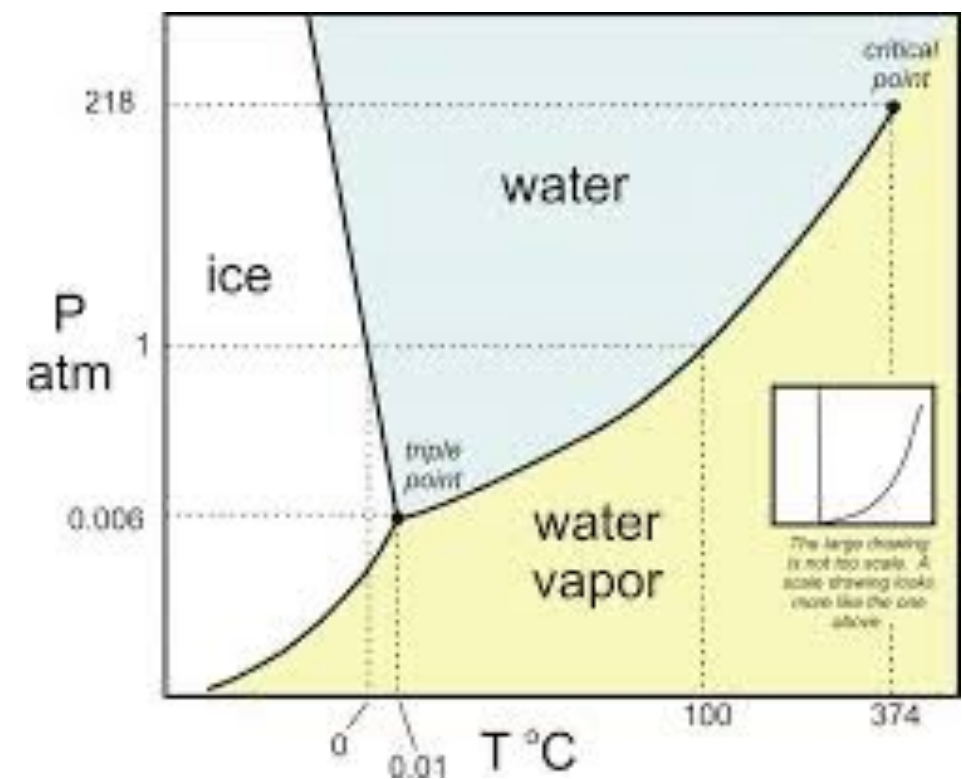
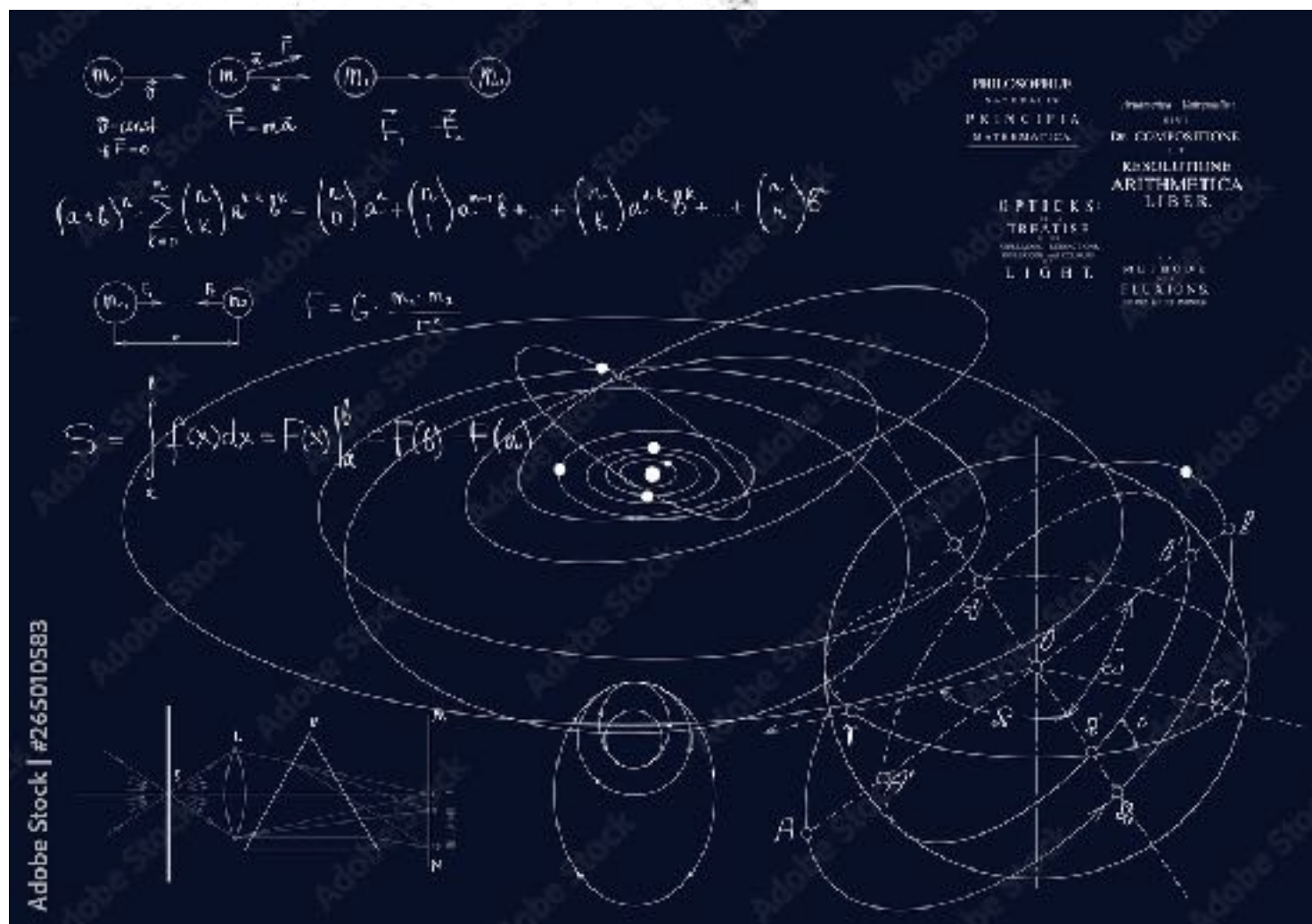


$$\oint \mathbf{E} \cdot d\mathbf{A} = q / \epsilon_0$$

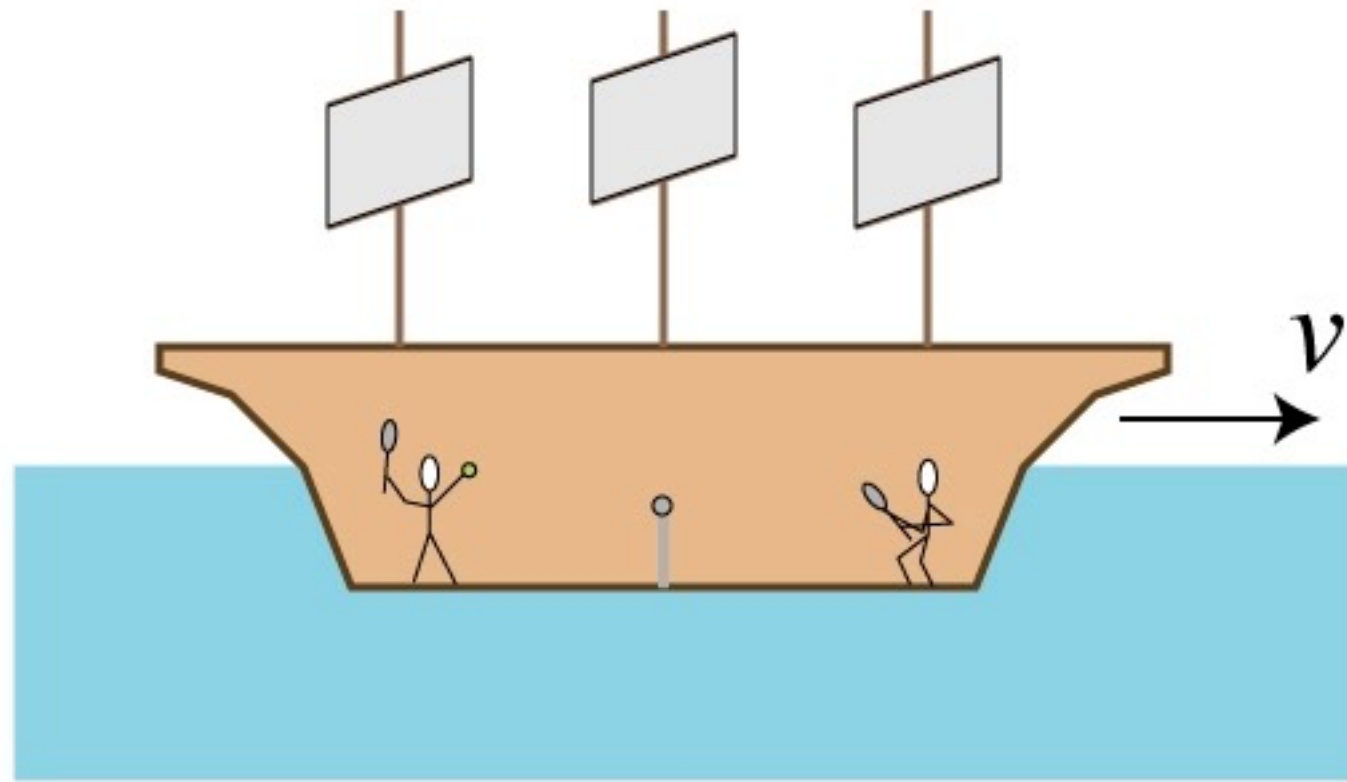
$$\oint \mathbf{B} \cdot d\mathbf{A} = 0$$

$$\oint \mathbf{E} \cdot d\mathbf{S} = -d\Phi_B / dt$$

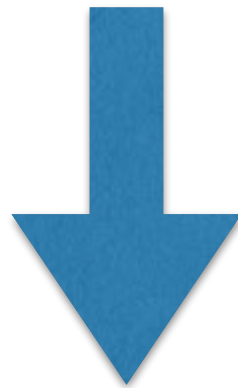
$$\oint \mathbf{B} \cdot d\mathbf{S} = \mu_0 i + \mu_0 \epsilon_0 d\Phi_E / dt$$



Galilean relativity

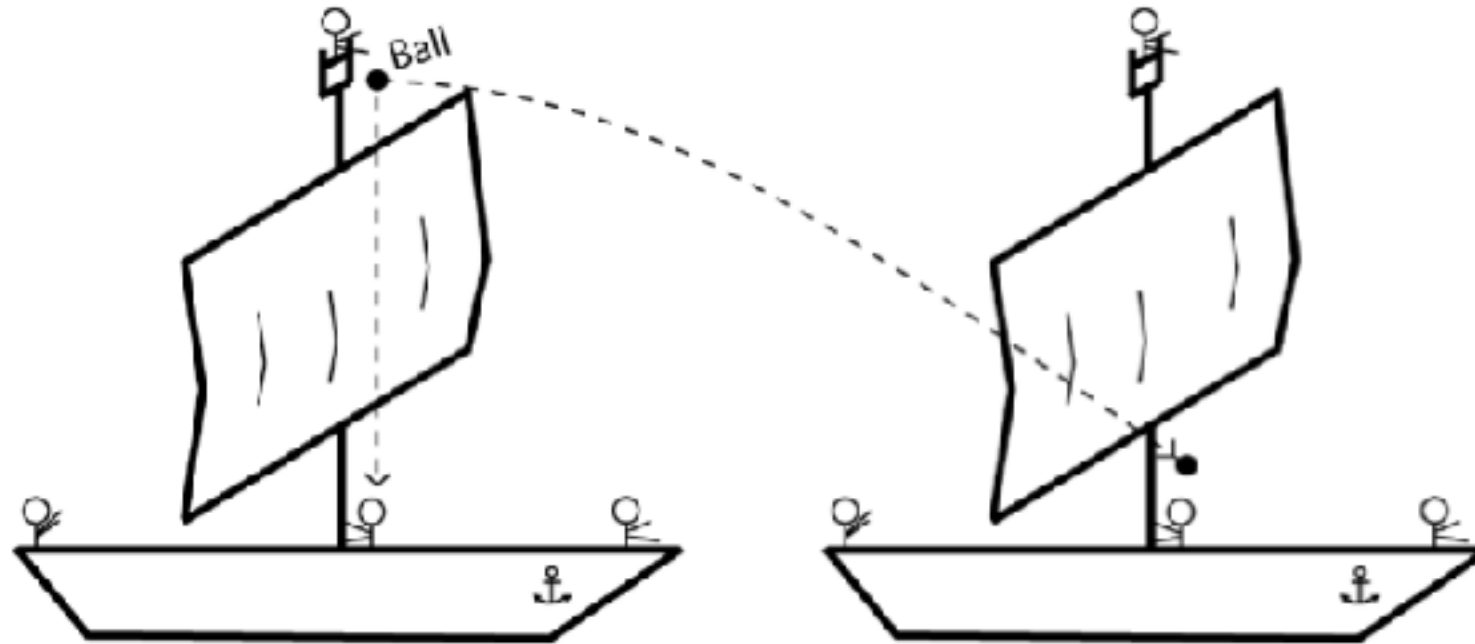


“The laws of physics are the same in all universally moving frames”



Time is absolute (the same for every one)

Galilean relativity



Change of frame:

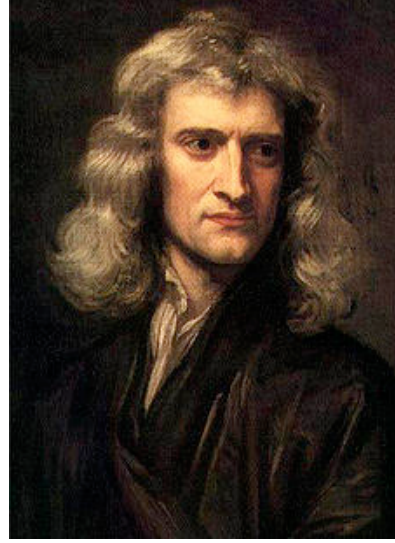
$$t = t'$$

$$\vec{x}' = \vec{x} + \vec{v}t$$

Speeds add:

$$\vec{v}' = \vec{v} + \vec{u}$$

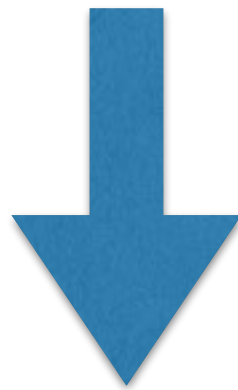
Newtonian relativity



G.G: “The laws of physics are the same in all
universally moving frames”

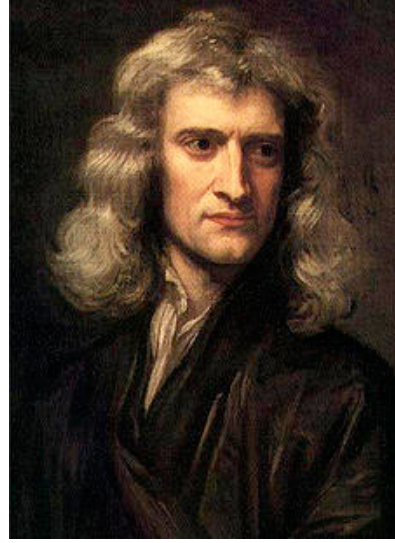
I.N.: “Universally moving with respect to what?”

- i. A body remains at constant speed unless acted upon by an external force
- ii. $F=ma$
- iii. Every action has an equal and opposite reaction



Time and space are absolute (the same for every one)

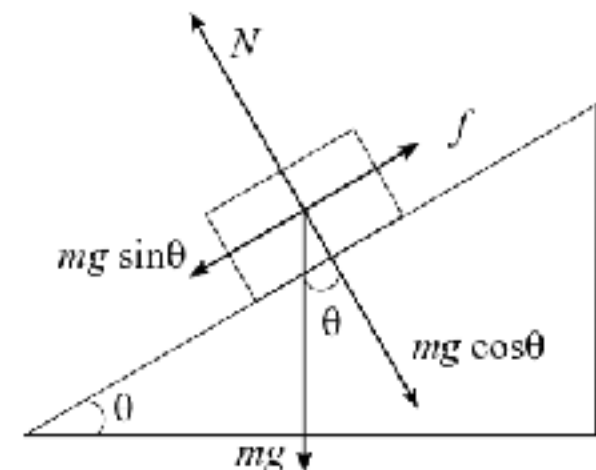
Newtonian relativity



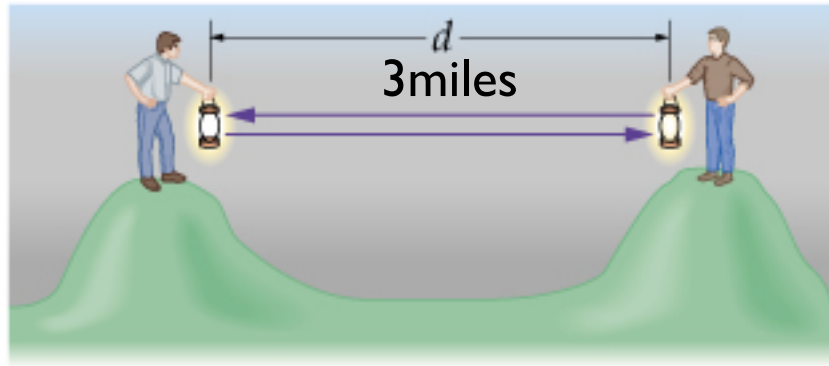
- i. A body remains at constant speed unless acted upon by an external force

An **inertial frame** is one where Newton's first law is true

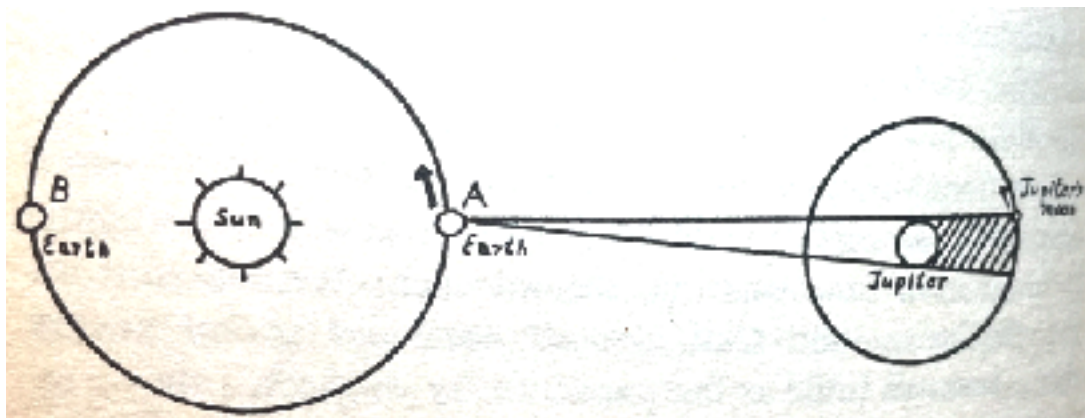
Physics is the same in all inertial frames



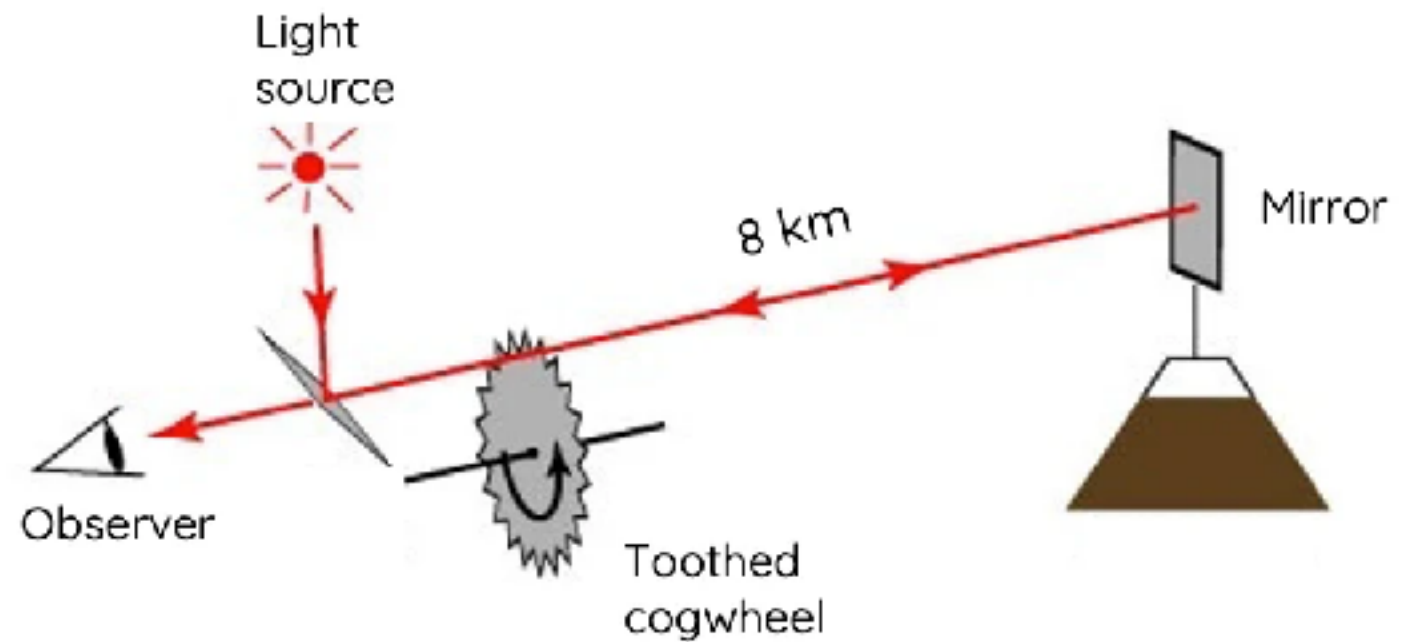
Measuring the speed of light



Galileo, c 1600

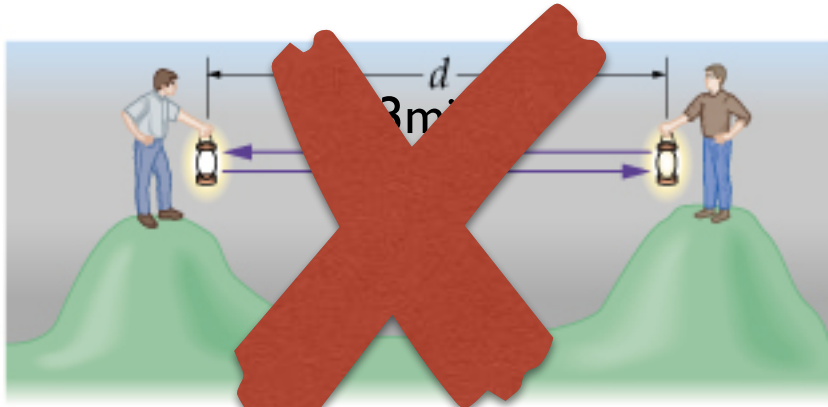


Ole Rømer, 1676

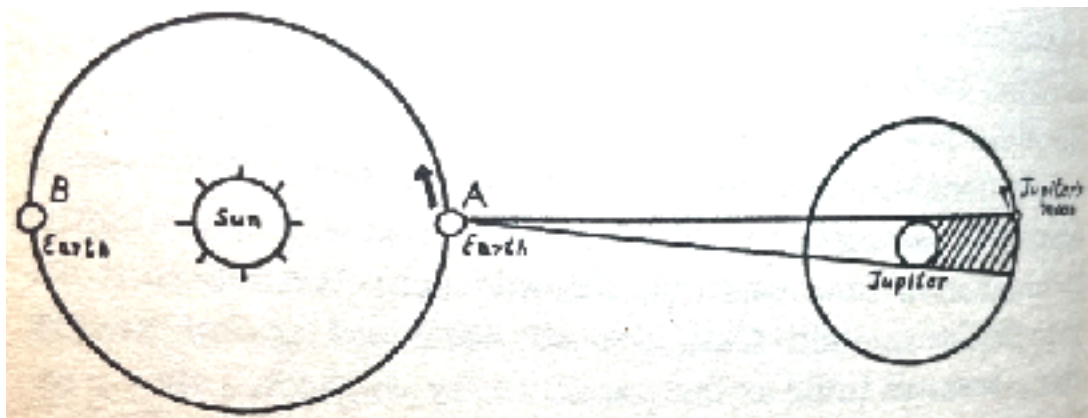


Armand Fizeau, 1849

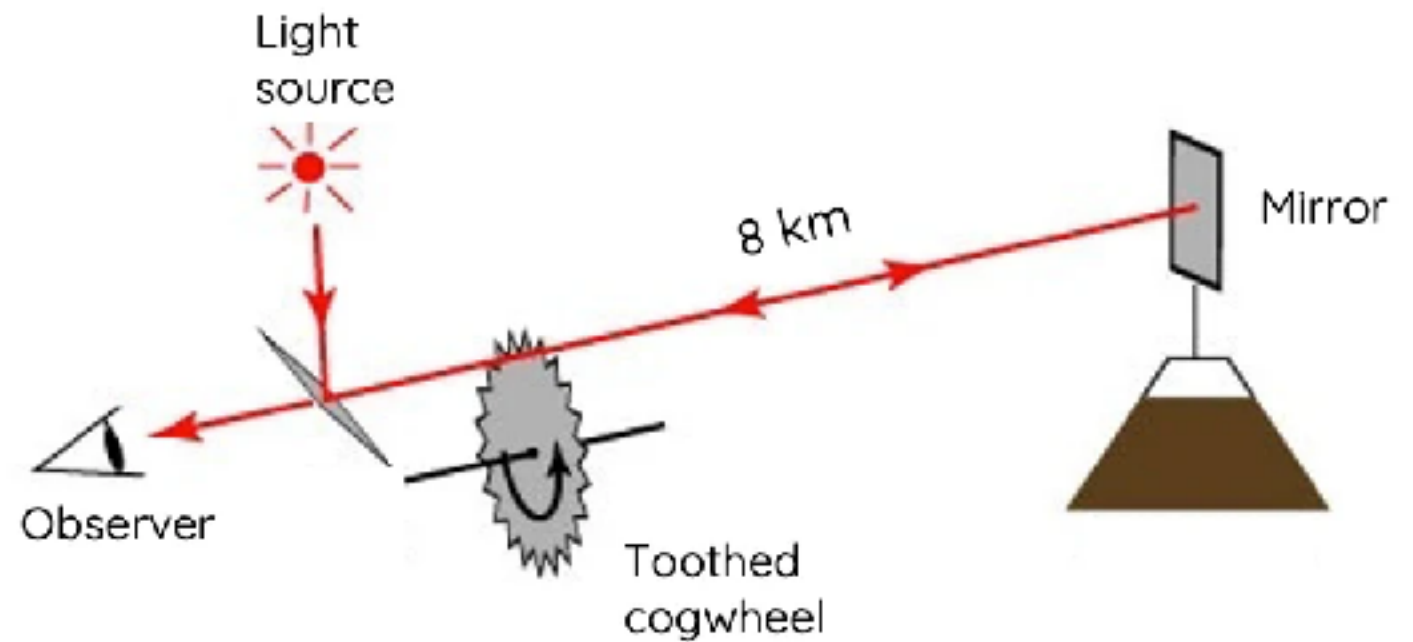
Measuring the speed of light



Galileo, c 1600

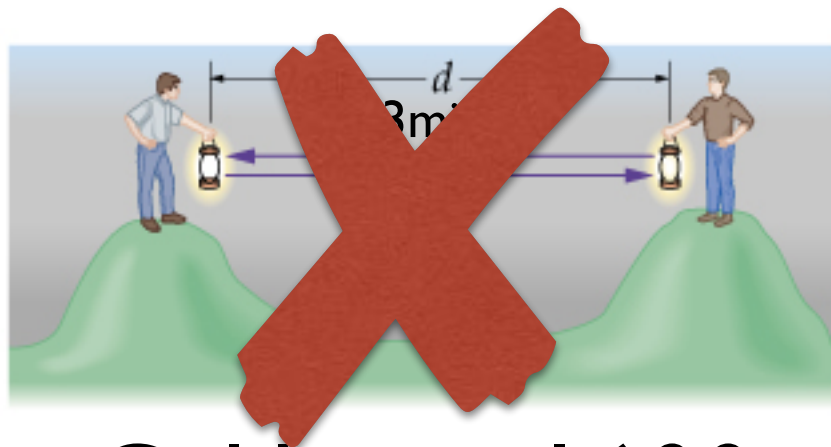


Ole Rømer, 1676



Armand Fizeau, 1849

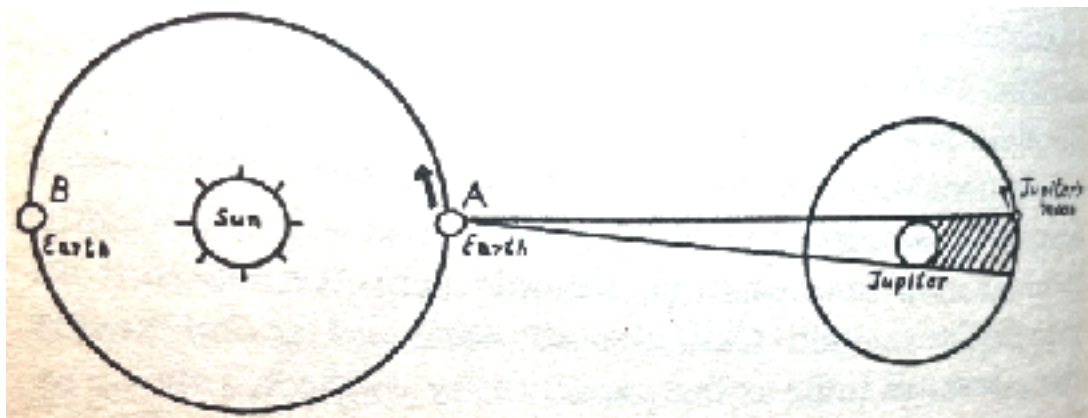
Measuring the speed of light



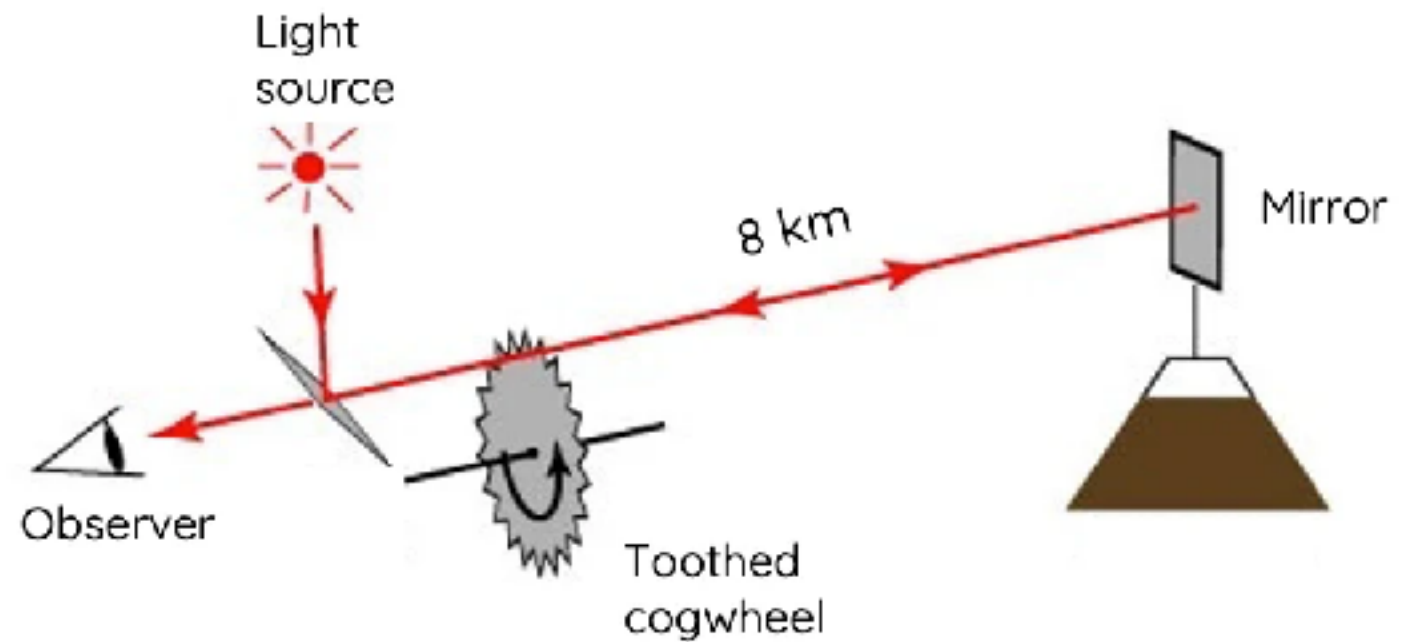
Galileo, c 1600



Newton, c 1686

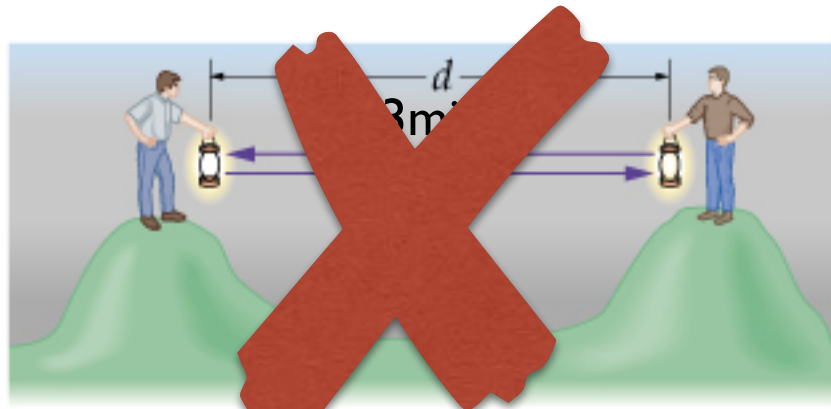


Ole Rømer, 1676



Armand Fizeau, 1849

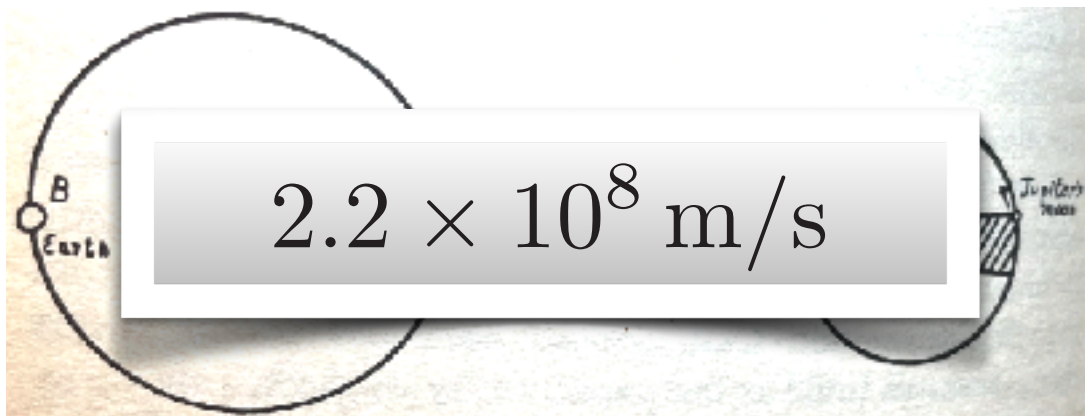
Measuring the speed of light



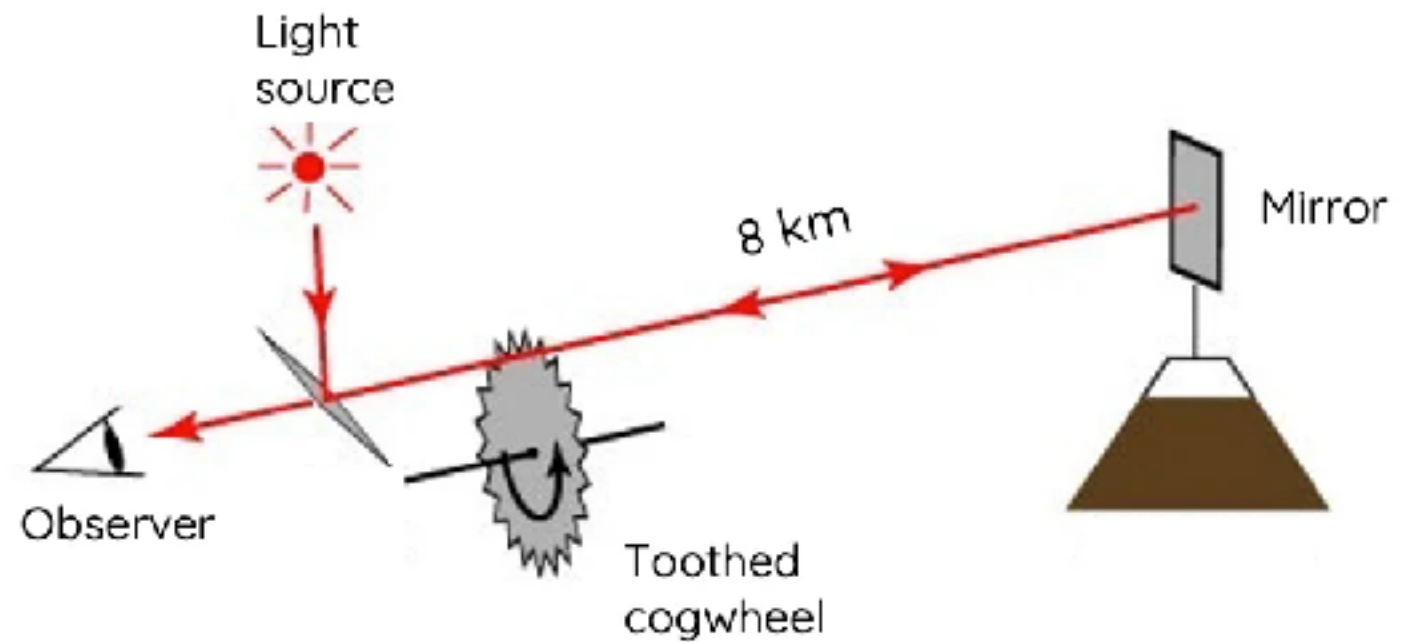
Galileo, c 1600



Newton, c 1686

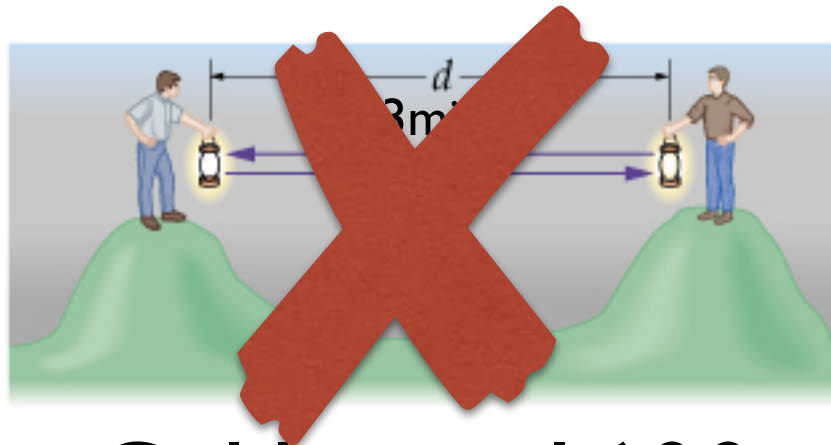


Ole Rømer, 1676



Armand Fizeau, 1849

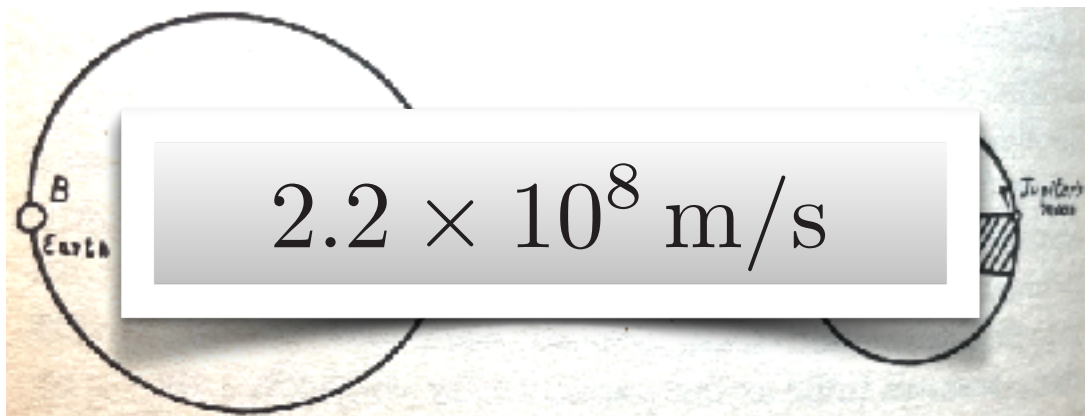
Measuring the speed of light



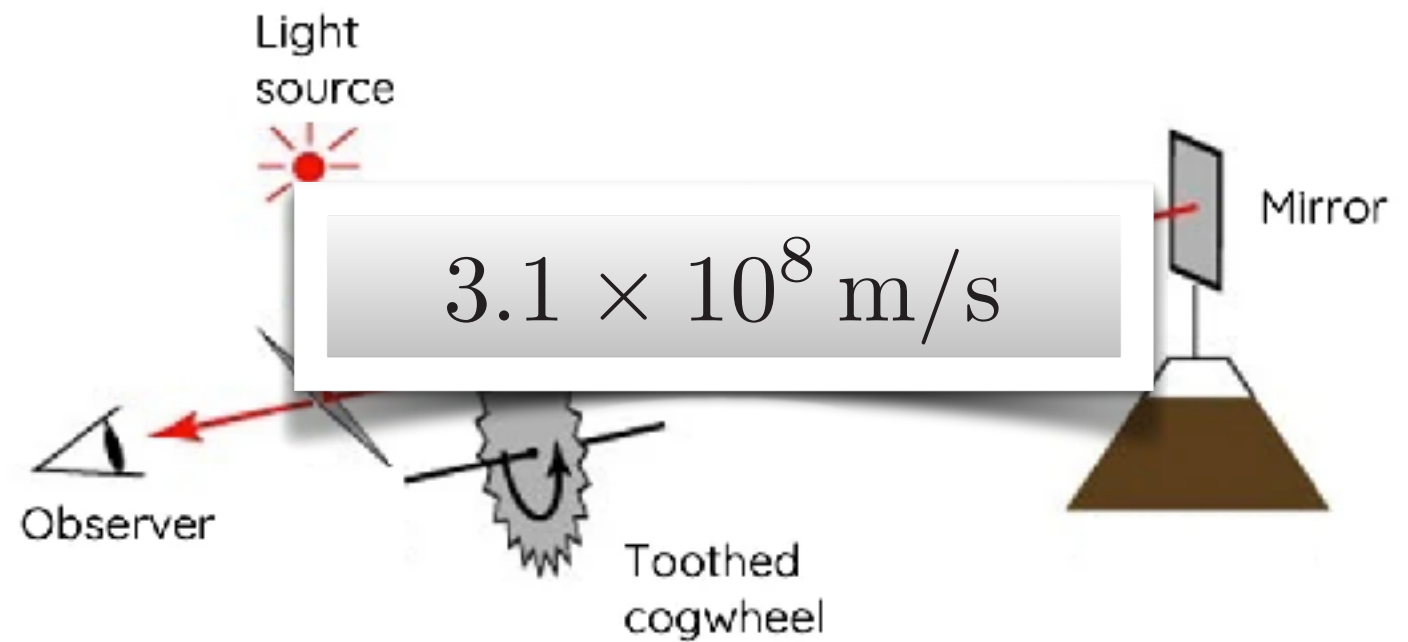
Galileo, c 1600



Newton, c 1686



Ole Rømer, 1676



Armand Fizeau, 1849

Maxwell's Equations



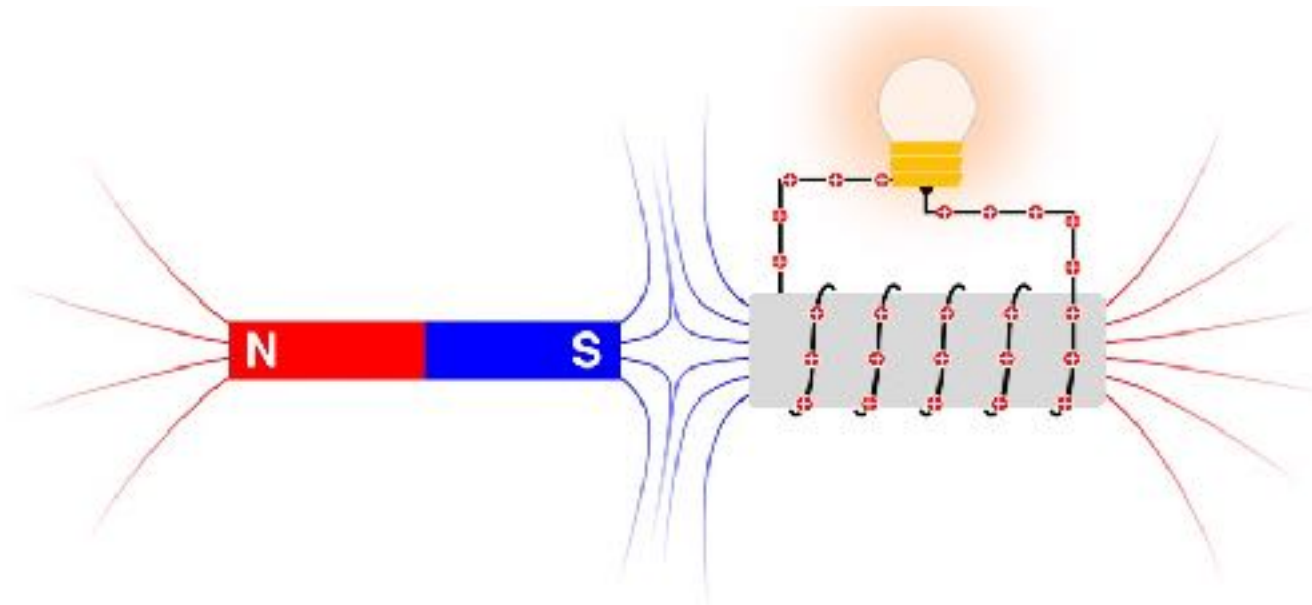
1864

$$\nabla \cdot \mathbf{E} = \frac{\rho}{\epsilon_0}$$

$$\nabla \cdot \mathbf{B} = 0$$

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$

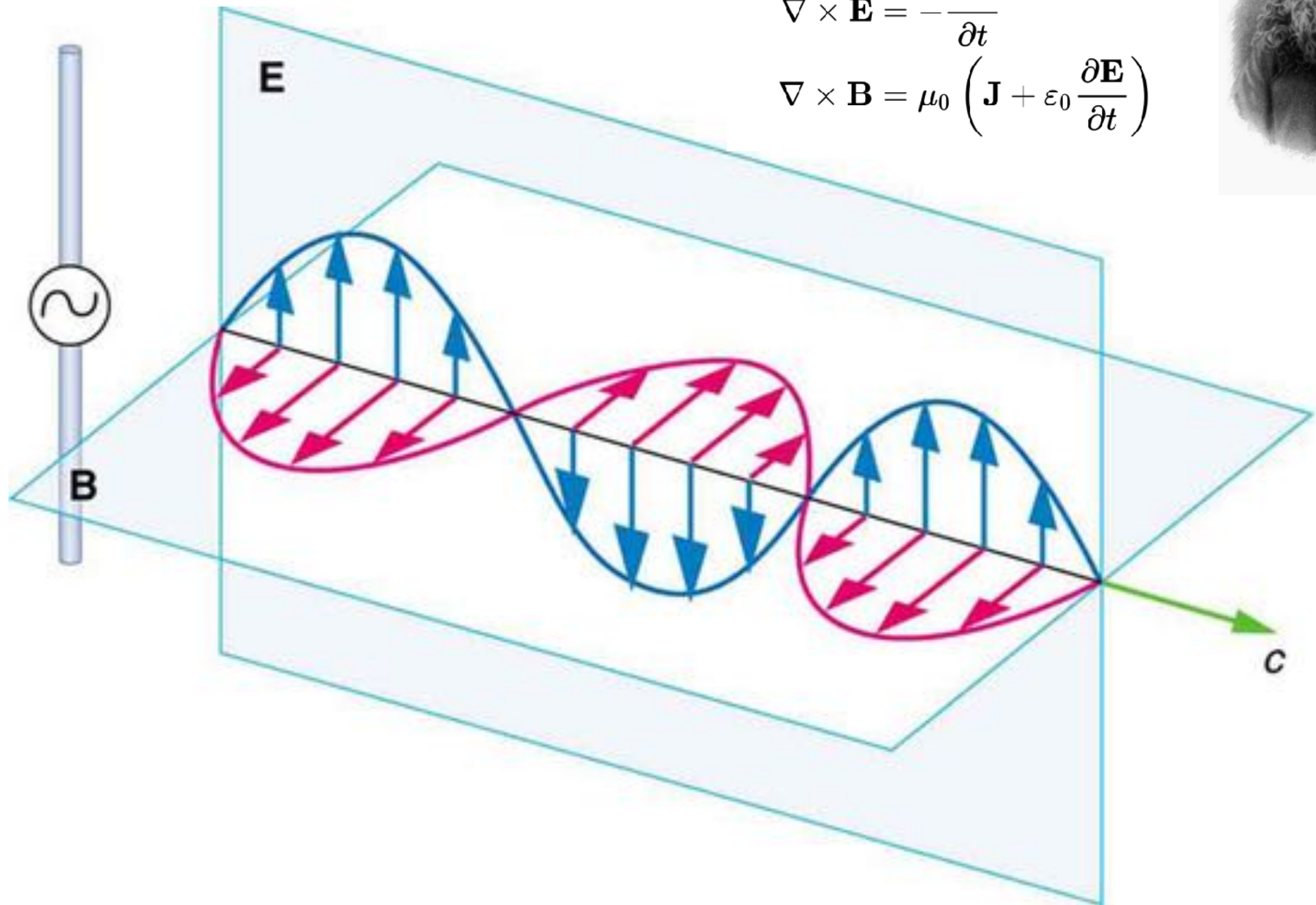
$$\nabla \times \mathbf{B} = \mu_0 \left(\mathbf{J} + \epsilon_0 \frac{\partial \mathbf{E}}{\partial t} \right)$$



Speed of light is fixed $c = \frac{1}{\sqrt{\epsilon_0 \mu_0}}$

Maxwell's Equations

$$\begin{aligned}\nabla \cdot \mathbf{E} &= \frac{\rho}{\epsilon_0} \\ \nabla \cdot \mathbf{B} &= 0 \\ \nabla \times \mathbf{E} &= -\frac{\partial \mathbf{B}}{\partial t} \\ \nabla \times \mathbf{B} &= \mu_0 \left(\mathbf{J} + \epsilon_0 \frac{\partial \mathbf{E}}{\partial t} \right)\end{aligned}$$



Accelerating charges can produce EM waves
Travel at the speed of light

Maxwell's Equations

$$\nabla \cdot \mathbf{E} = \frac{\rho}{\epsilon_0}$$



ELECTROMAGNETIC SPECTRUM



RADIO WAVES



MICROWAVES



INFRARED



VISIBLE



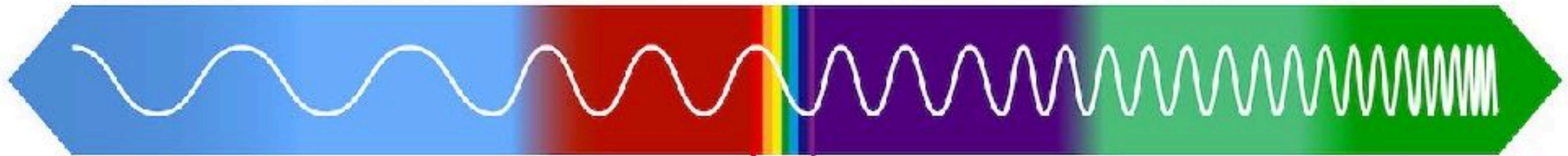
ULTRAVIOLET



X-RAYS



GAMMA RAYS



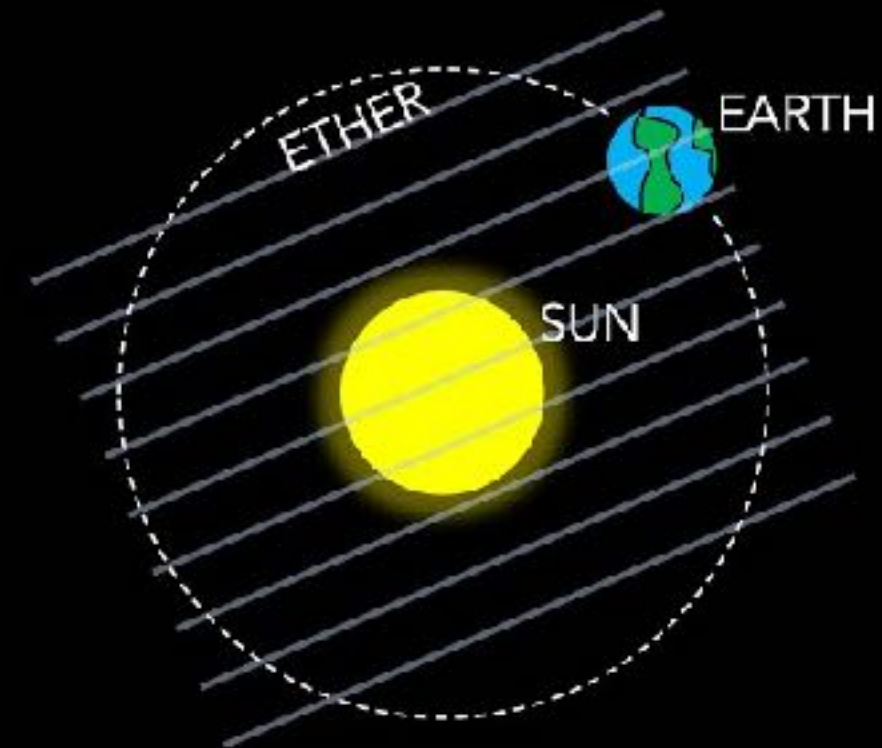
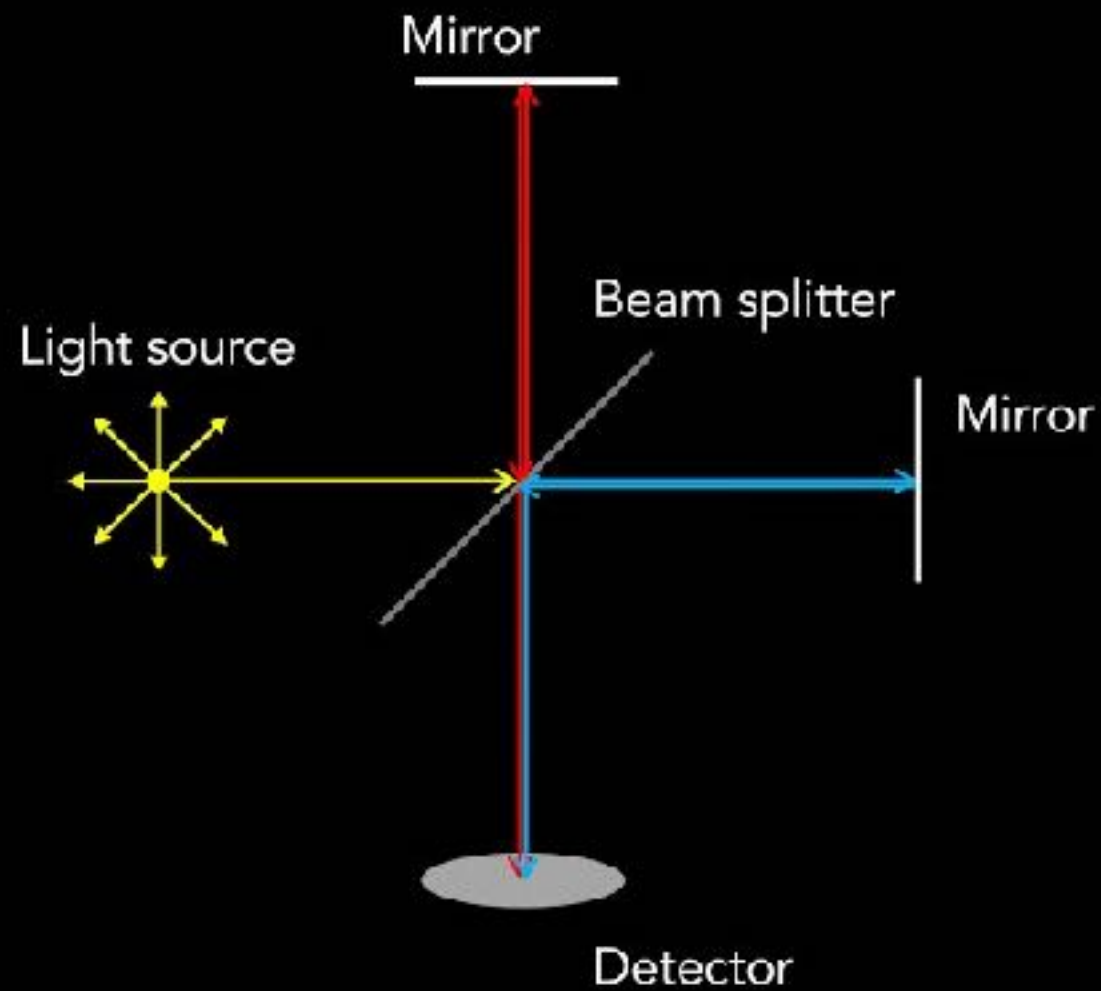
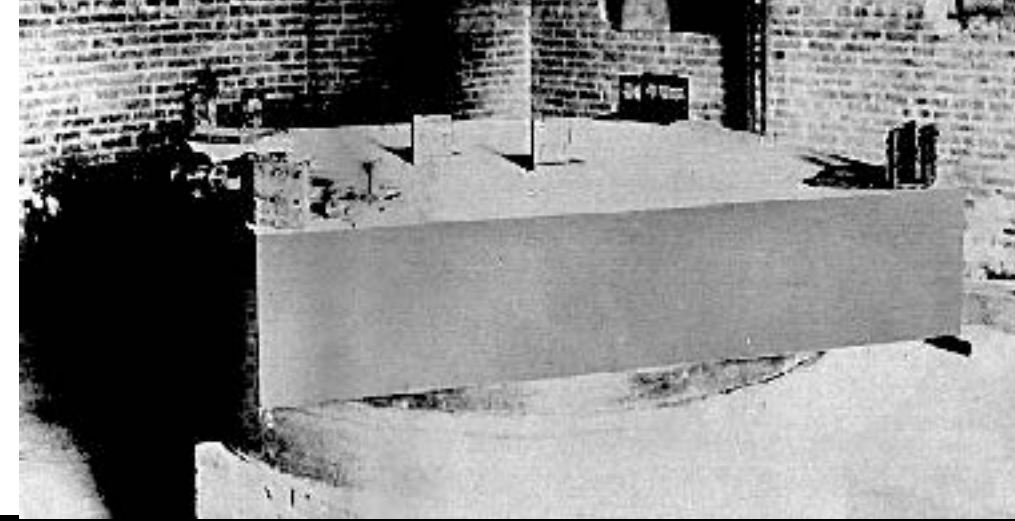
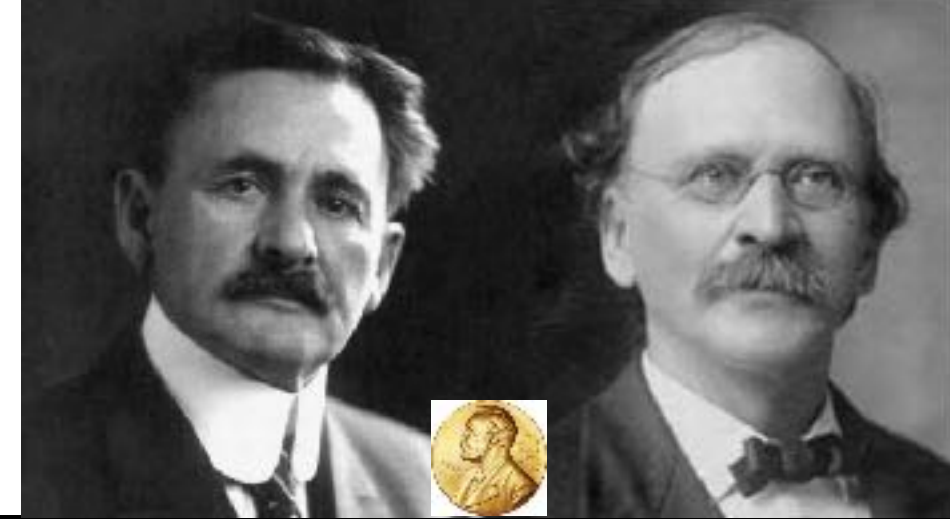
Accelerating charges can produce EM waves
Travel at the speed of light

Maxwell's Equations

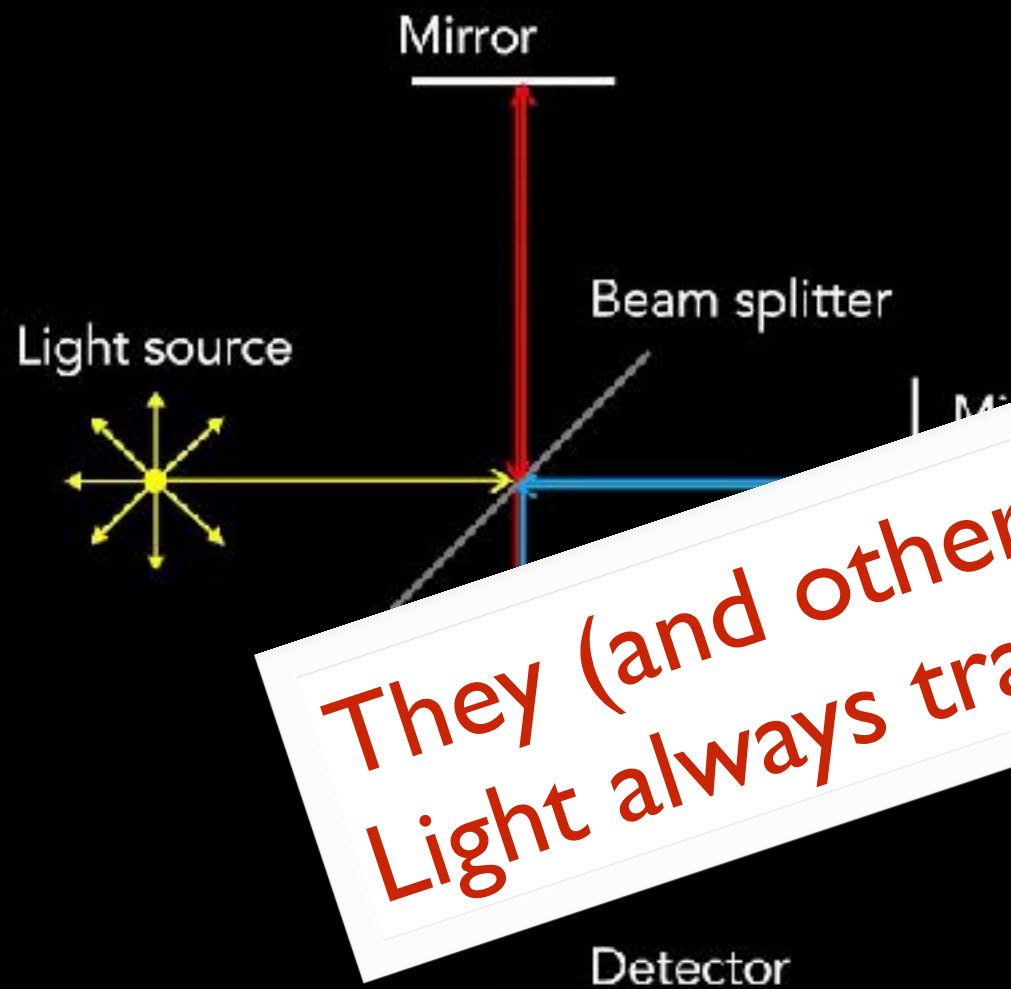
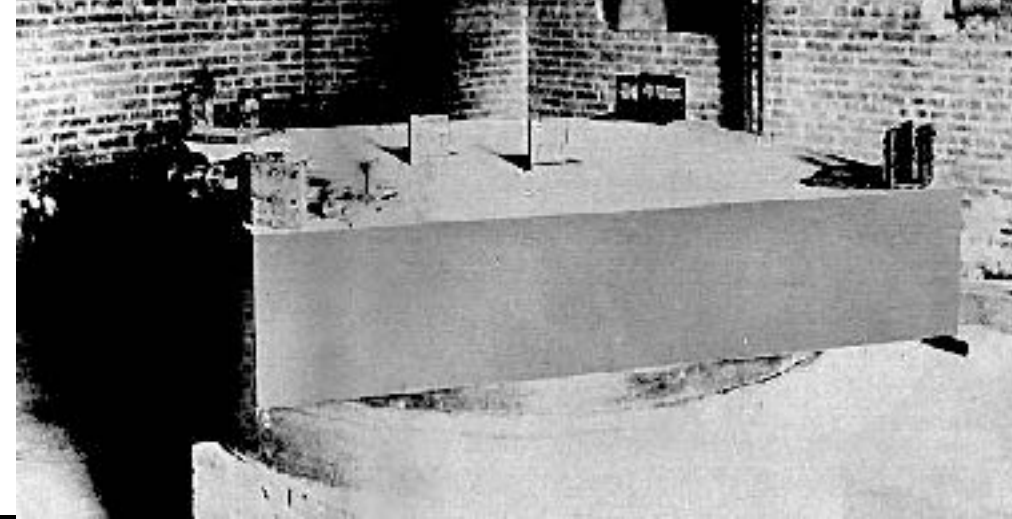
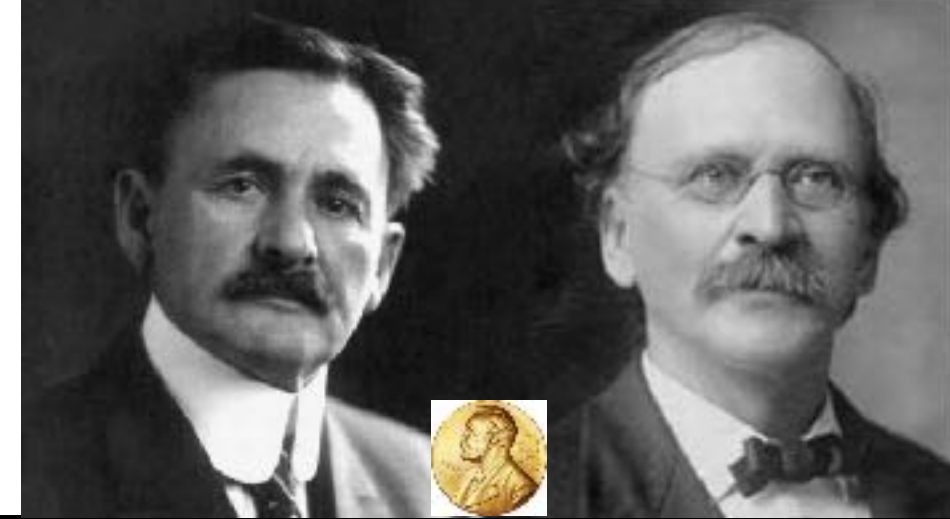


- Water waves wiggle on water
- Sound waves move through air/material
- What is wiggling in EM radiation?

Luminiferous Aether



The Michelson-Morley Experiment



They (and others) failed to find the aether!
Light always travels at $c \approx 3 \times 10^8 \text{ m/sec}$

The Michelson-
Morley
Experiment

Lorentz and Poincare and the symmetries of Maxwell's Equations

$$\nabla \cdot \mathbf{E} = \frac{\rho}{\epsilon_0}$$

$$\nabla \cdot \mathbf{B} = 0$$

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$

$$\nabla \times \mathbf{B} = \mu_0 \left(\mathbf{J} + \epsilon_0 \frac{\partial \mathbf{E}}{\partial t} \right)$$

Lorentz discovered that Maxwell's equations are invariant under a strange transformation

$$x' = \gamma(x - vt)$$

$$t' = \gamma \left(t - \frac{vx}{c^2} \right)$$

Annus Mirabilis (1905)



Annus Mirabilis (1905)

1. Photoelectric effect 

2. Brownian motion

3. Special Relativity

4. $E=mc^2$



Annus Mirabilis (1905)

1. Photoelectric effect 
2. Brownian motion
3. Special Relativity
4. $E=mc^2$



Two postulates:

1. Physics is the same in all inertial frames (you cannot detect the aether)
2. Speed of light is the same for all observers.

Annus Mirabilis (1905)

1. Photoelectric effect 
2. Brownian motion
3. Special Relativity
4. $E=mc^2$

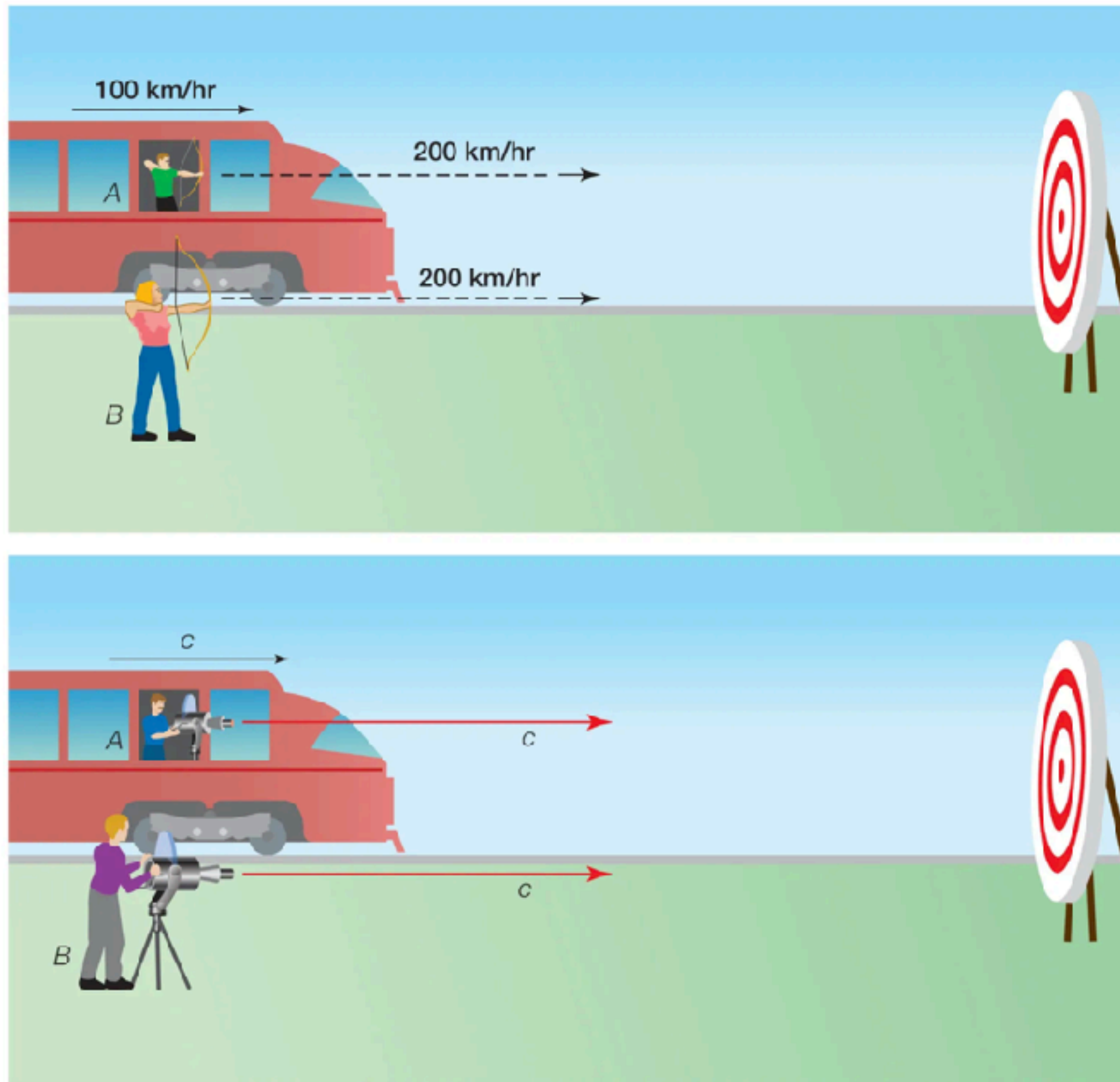


Two postulates:

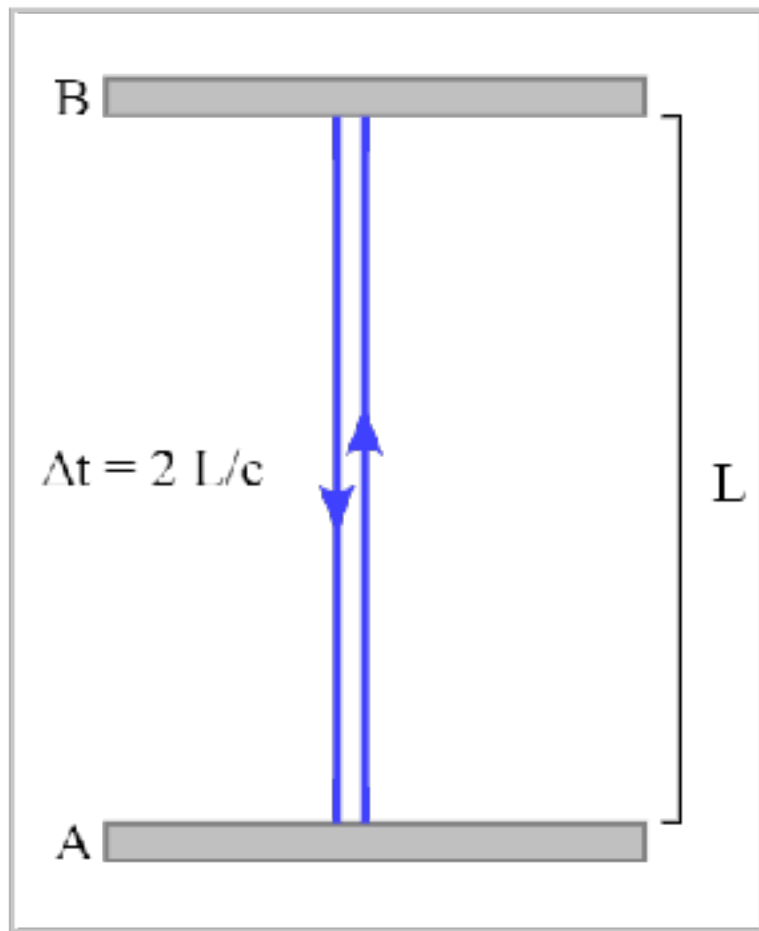
1. Physics is the same in all inertial frames (you cannot detect the aether)
2. Speed of light is the same for all observers.

There is no absolute space or time, all motion is relative

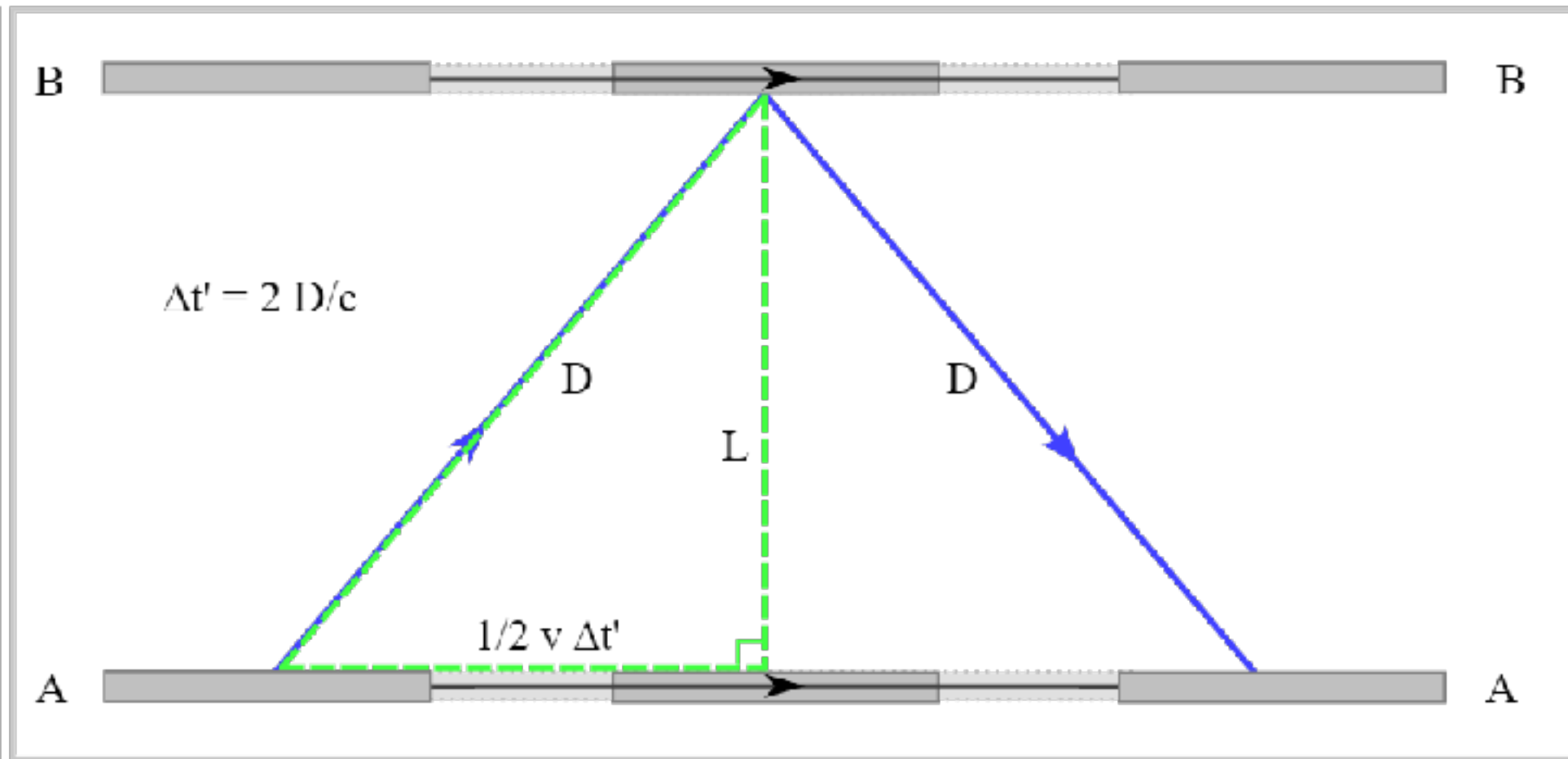
The second postulate is very counter intuitive



Time dilation — Gedanken Experiment

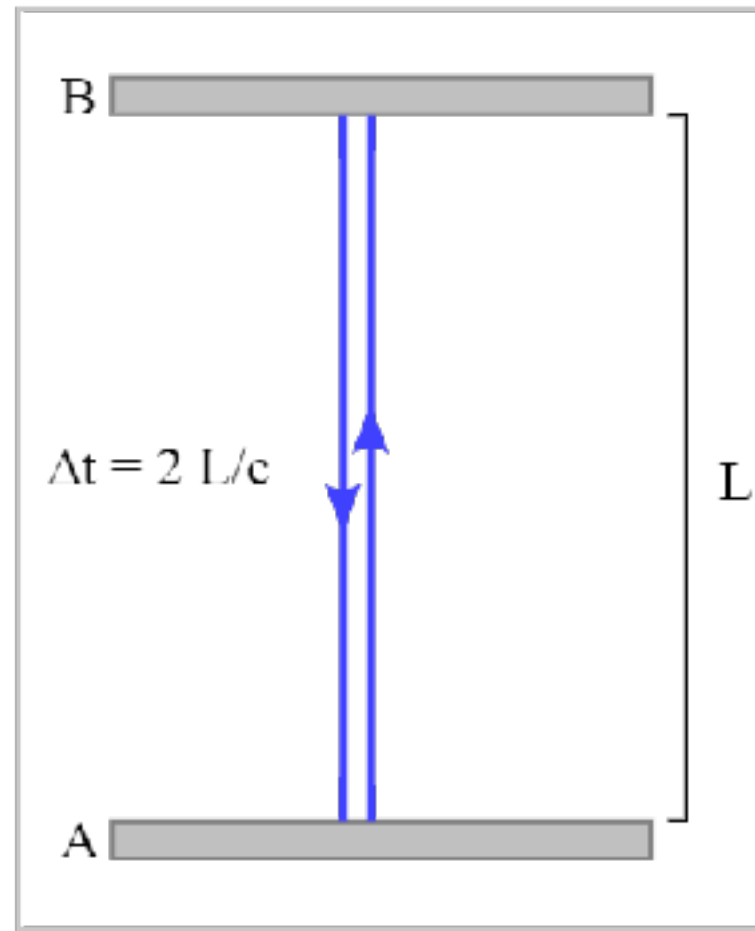


View from train

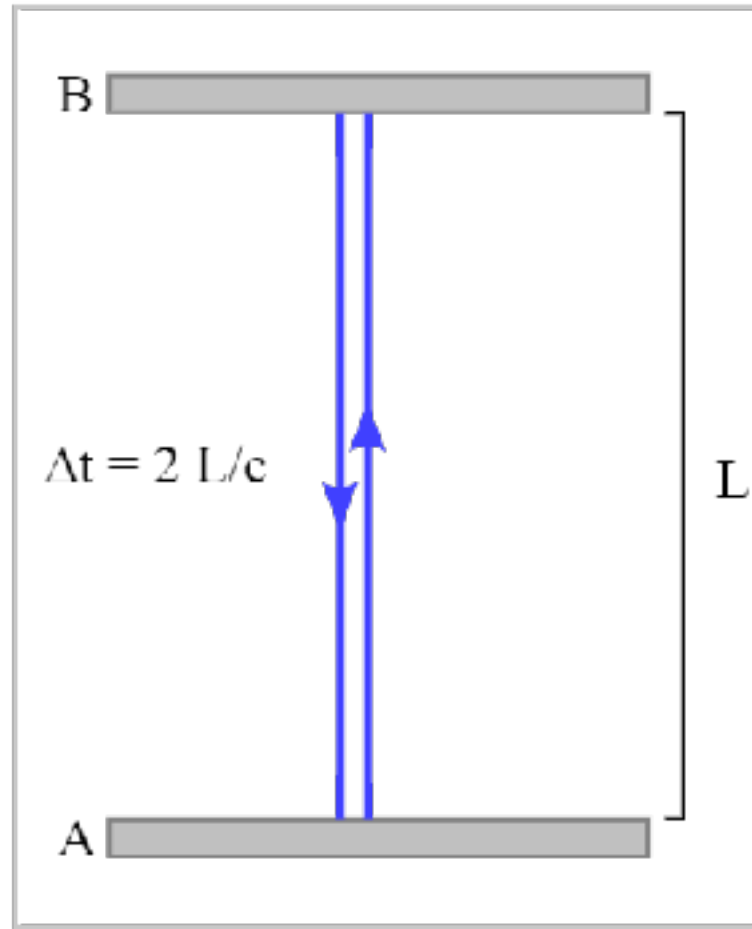


View from platform

Use light as a clock, we know that light always travels at c

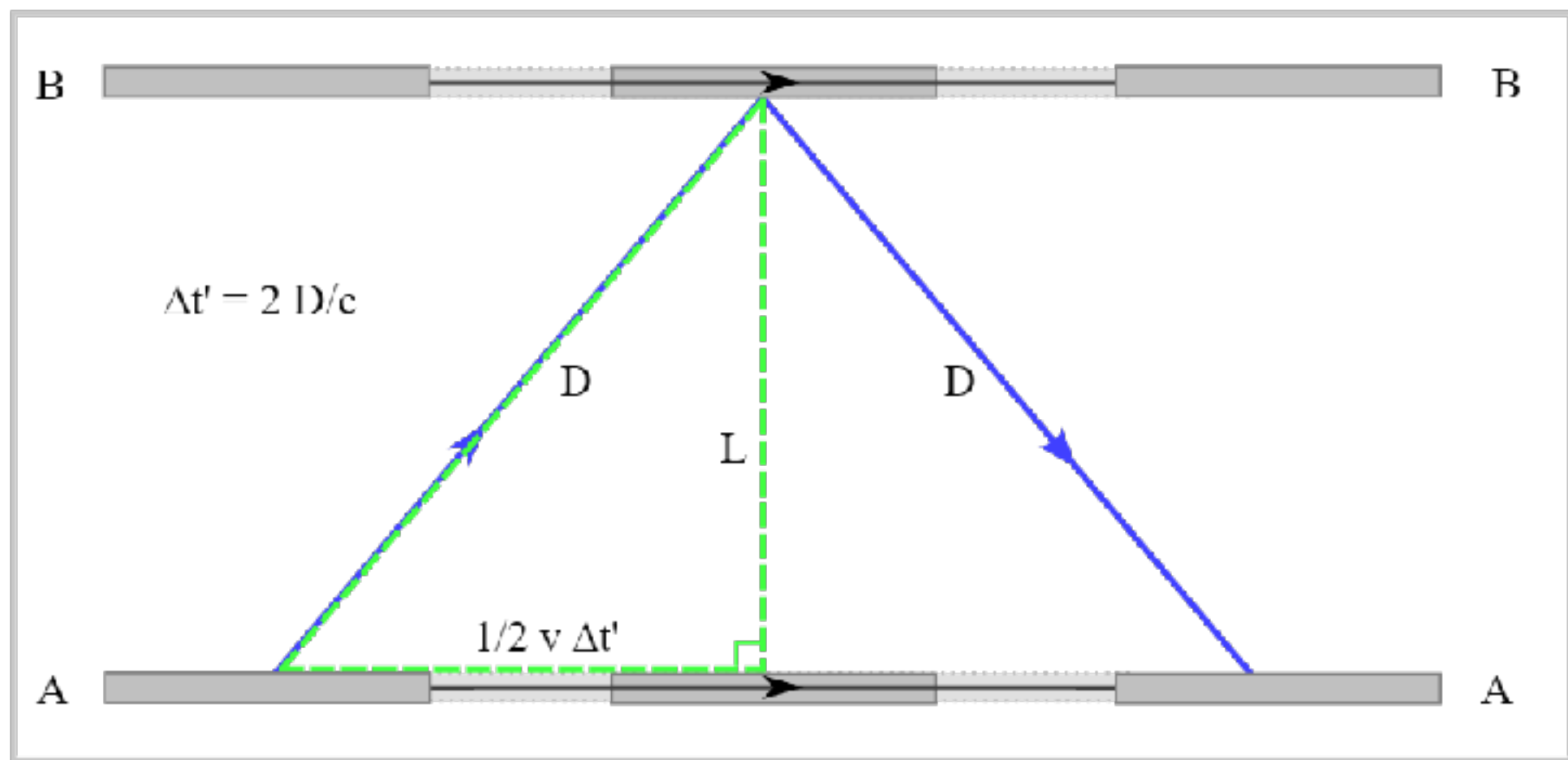


View from train



View from train

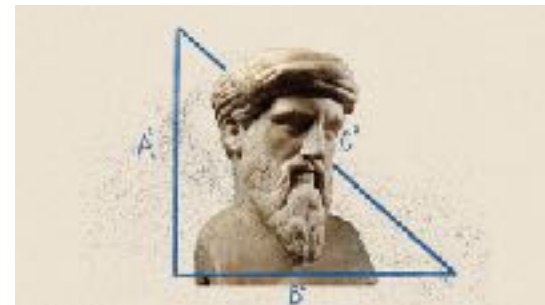
$$\Delta t = 2 \frac{L}{c}$$

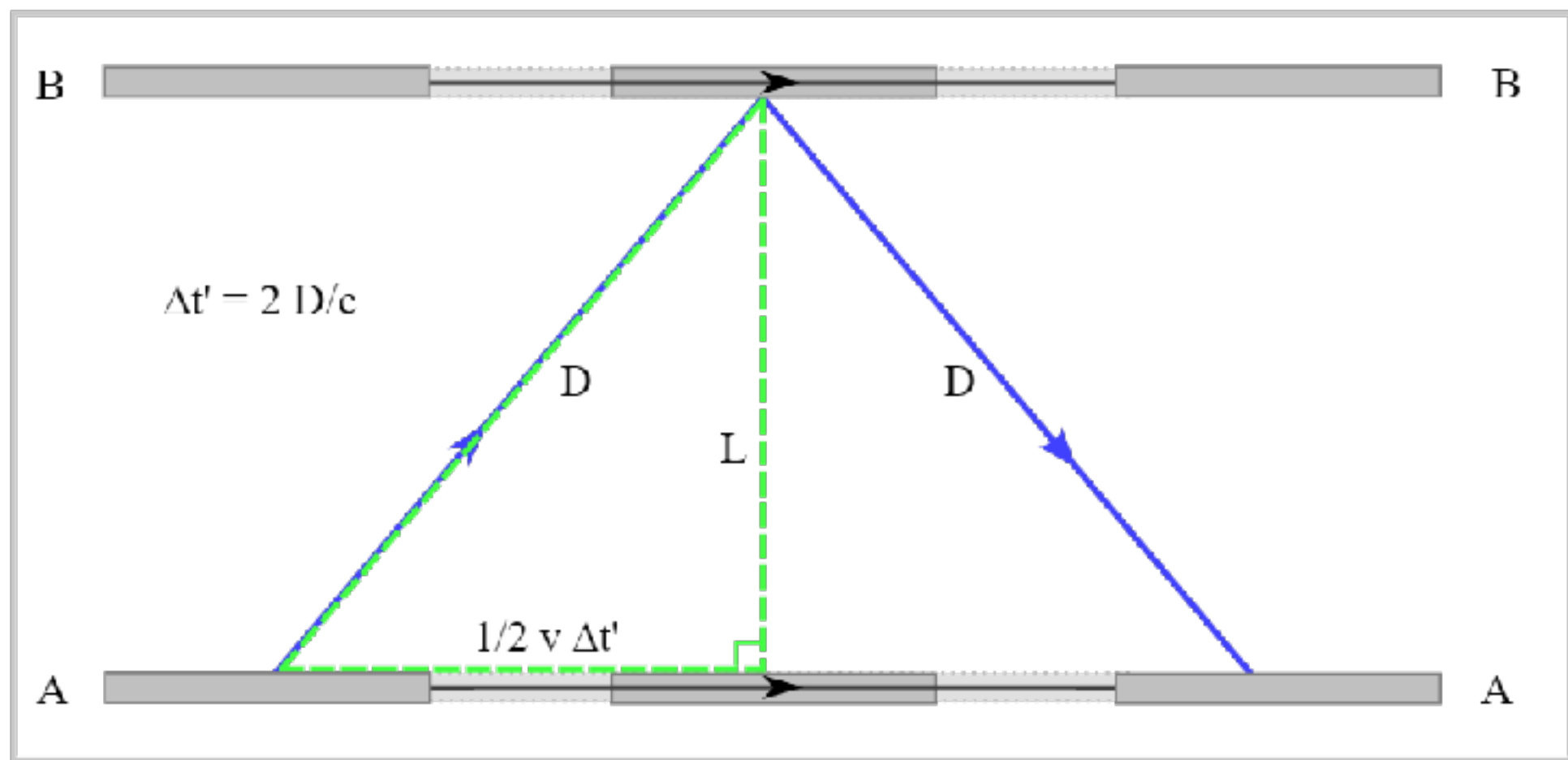


View from platform

Distance traveled:

$$D^2 = L^2 + \left(\frac{1}{2}v\Delta t'\right)^2$$

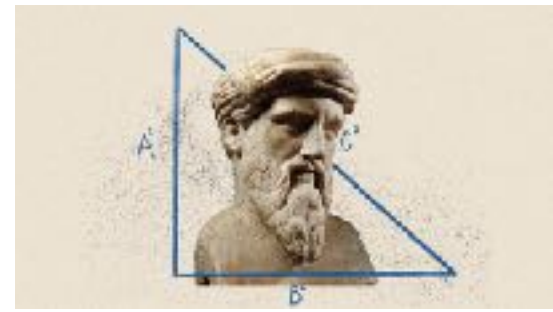


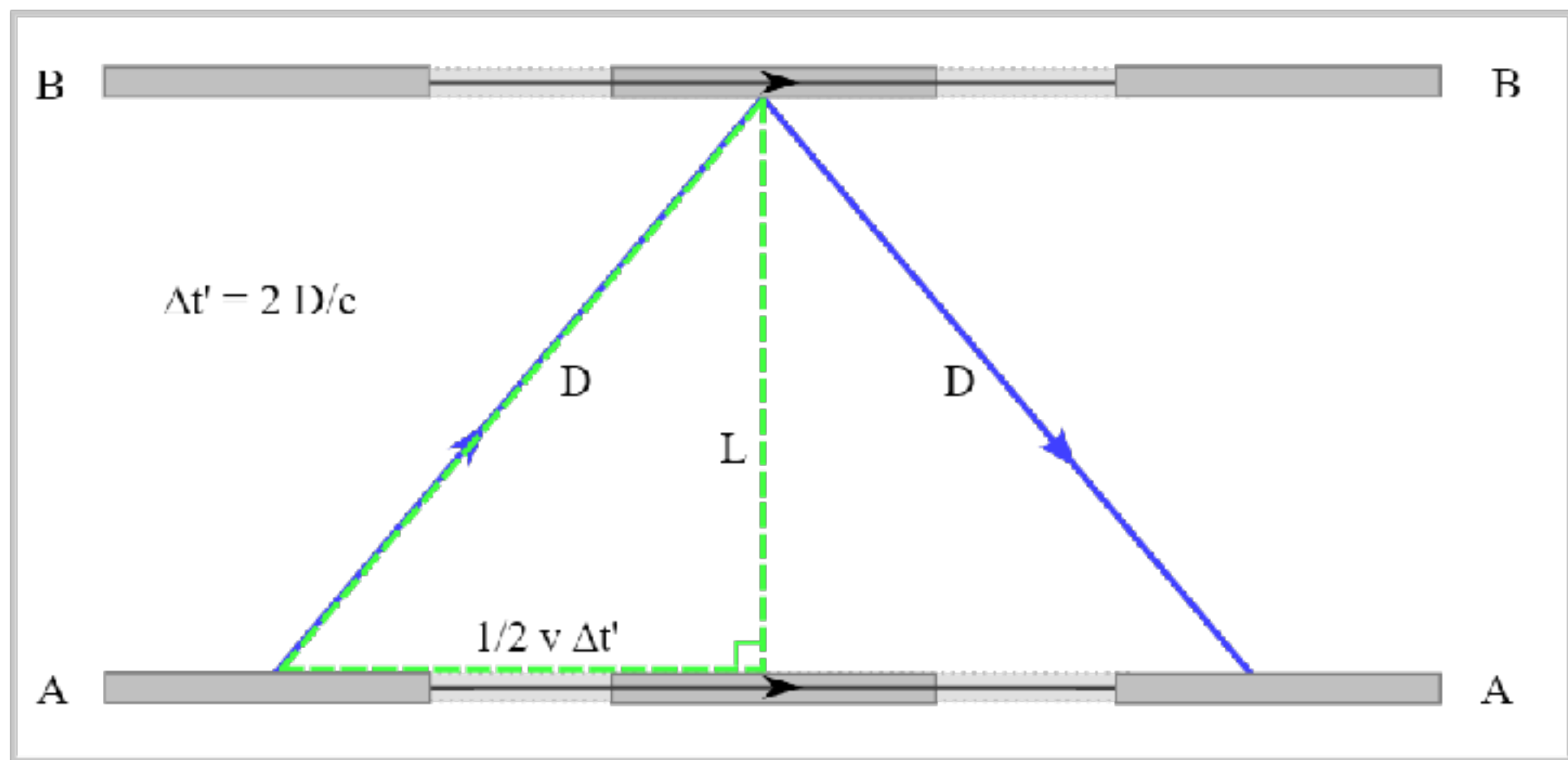


View from platform

Distance traveled: $D^2 = L^2 + \left(\frac{1}{2} v \Delta t'\right)^2$

$$D^2 = \left(\frac{c \Delta t}{2}\right)^2 \left[1 + \left(\frac{v}{c}\right)^2 \left(\frac{\Delta t'}{\Delta t}\right)^2\right]$$



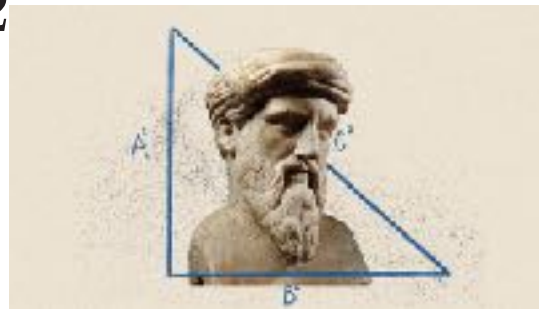


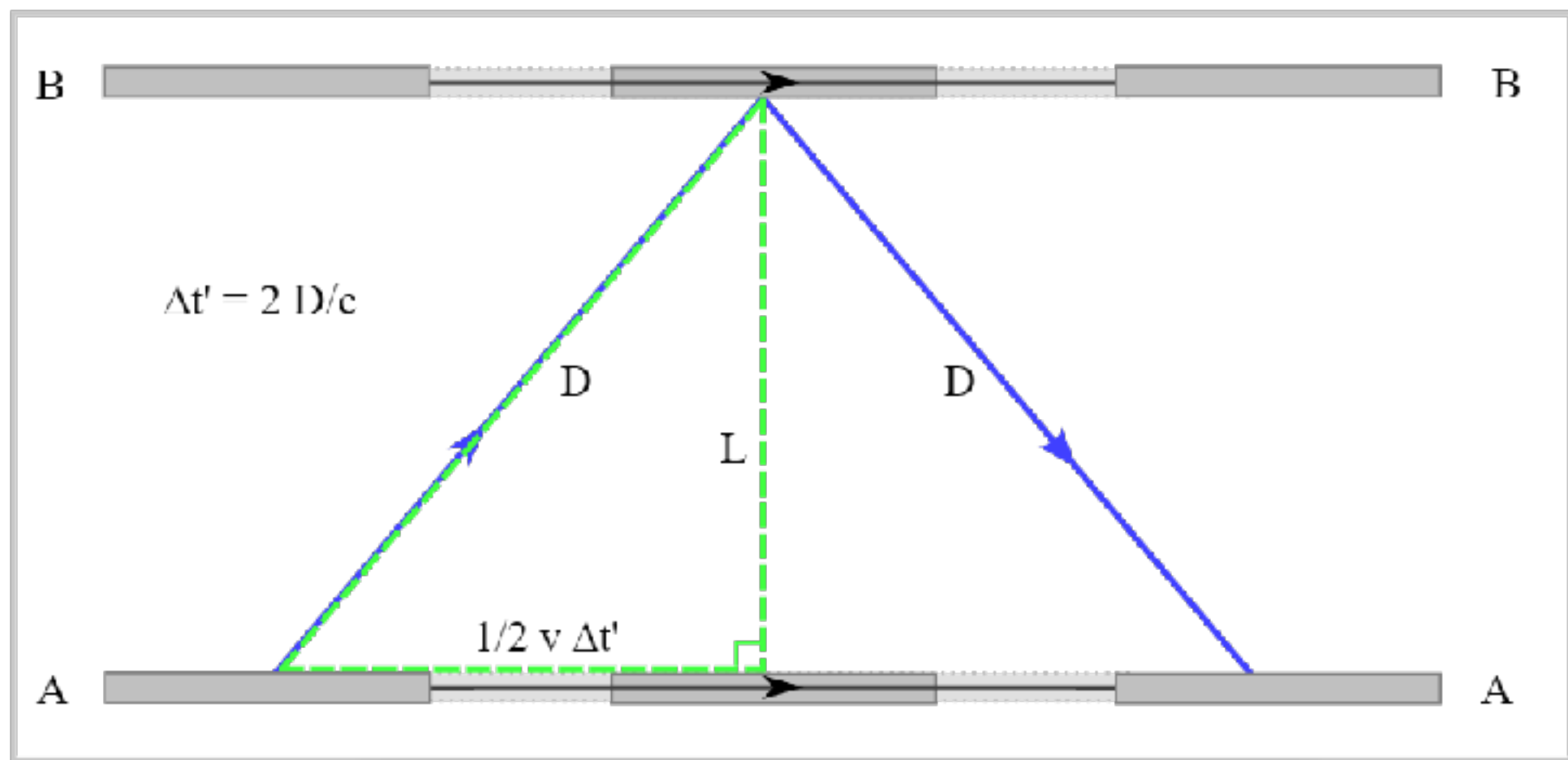
View from platform

Distance traveled: $D^2 = L^2 + \left(\frac{1}{2}v\Delta t'\right)^2$

$$D^2 = \left(\frac{c\Delta t}{2}\right)^2 \left[1 + \left(\frac{v}{c}\right)^2 \left(\frac{\Delta t'}{\Delta t}\right)^2\right]$$

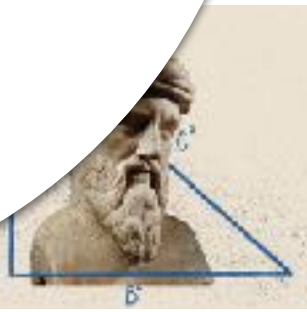
$$\Delta t' = \Delta t \left[1 + \left(\frac{v}{c}\right)^2 \left(\frac{\Delta t'}{\Delta t}\right)^2\right]^{1/2}$$





Distance

$$\Delta t' = \frac{\Delta t}{\sqrt{1 - \frac{v^2}{c^2}}} \equiv \gamma \Delta t$$

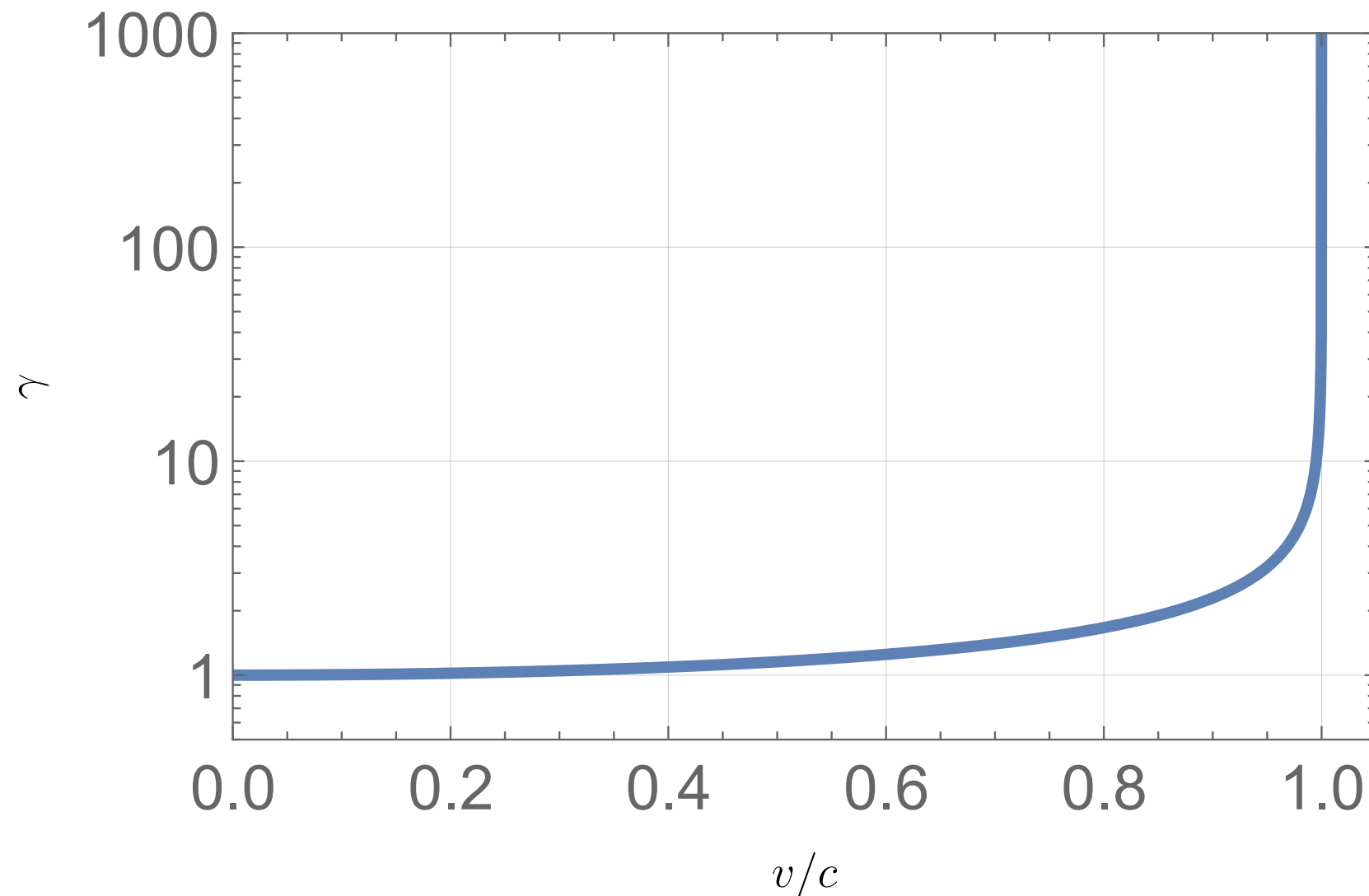


Time dilation — Gedanken Experiment

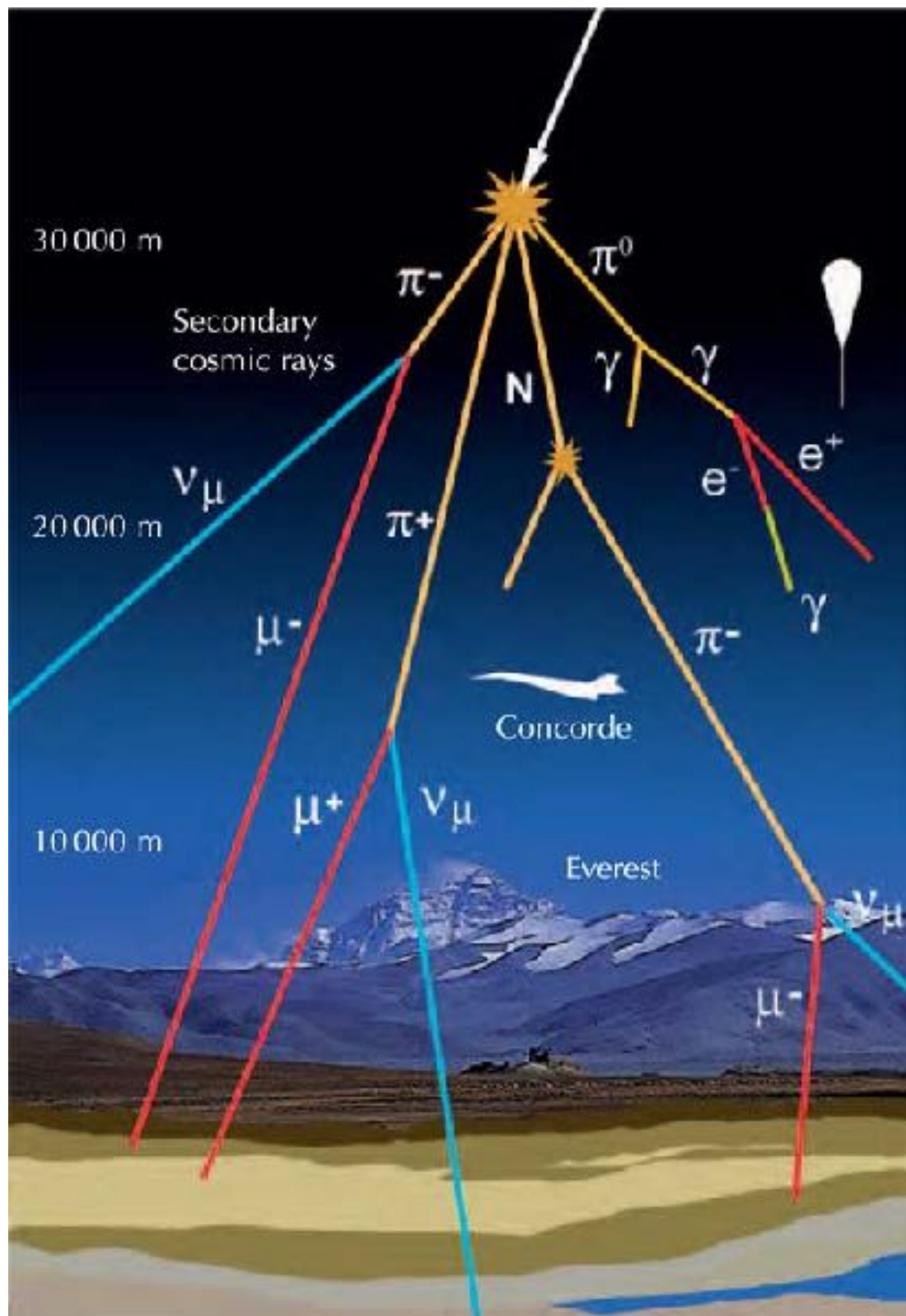
Moving clocks “tick slower”

c is a maximum speed for all things

Only really noticeable for v close to c



Time dilation — Real Experiment



$$\tau = 2.2\mu s$$

$$c\tau = 660\text{m}$$

$$\gamma c\tau \gg 660\text{m}$$

Frisch and Smith in 1962 measured flux of muons at different altitudes.

Rate, and energies, consistent with muons moving at $0.995c$.

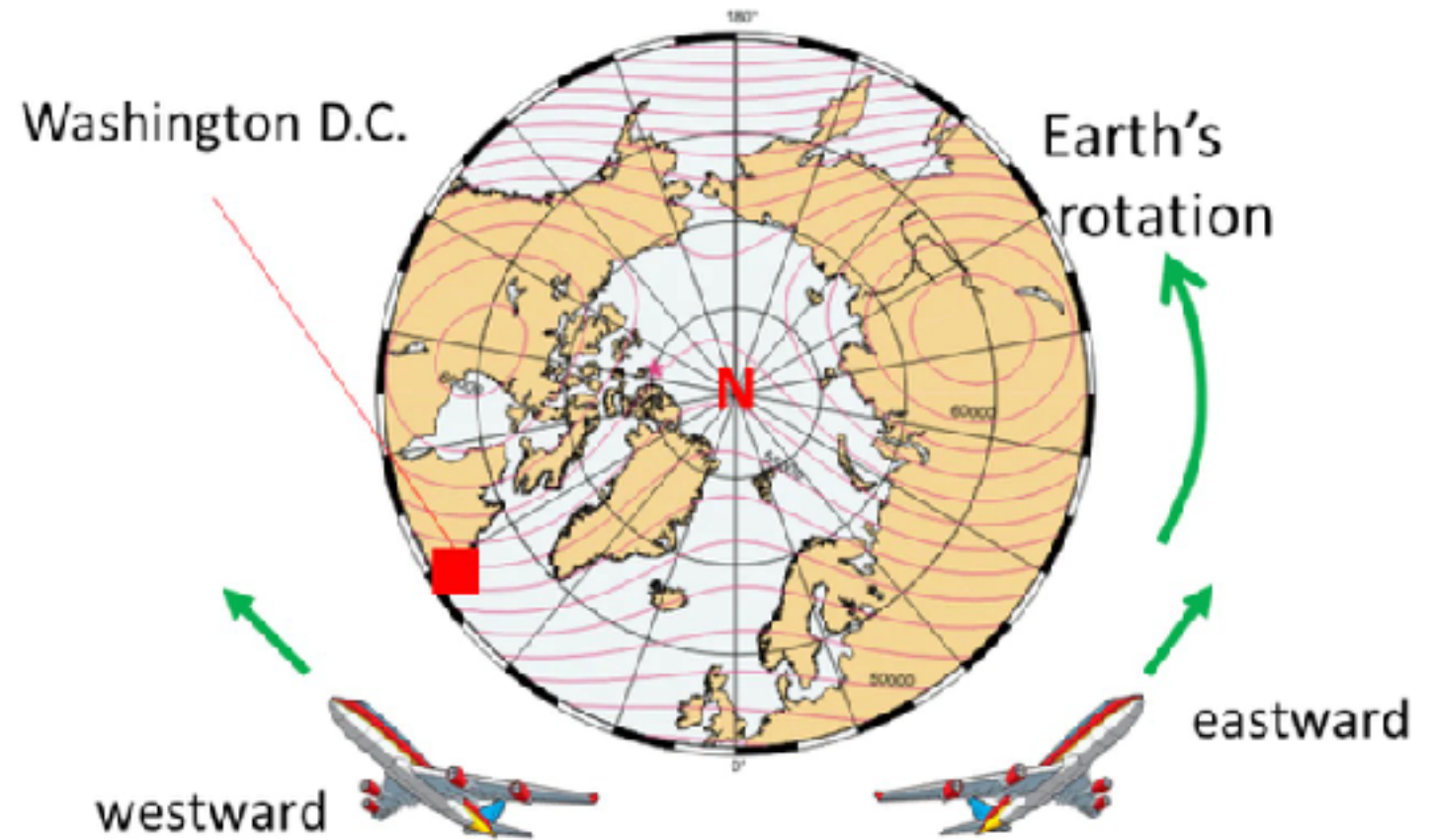
Variation with altitude what expected from time dilation

Time dilation — Real Experiment



Hafele and Keating

Speed of plane, relative to clocks on ground gave a 0.1 microsecond change, in agreement with relativity





INTERMISSION

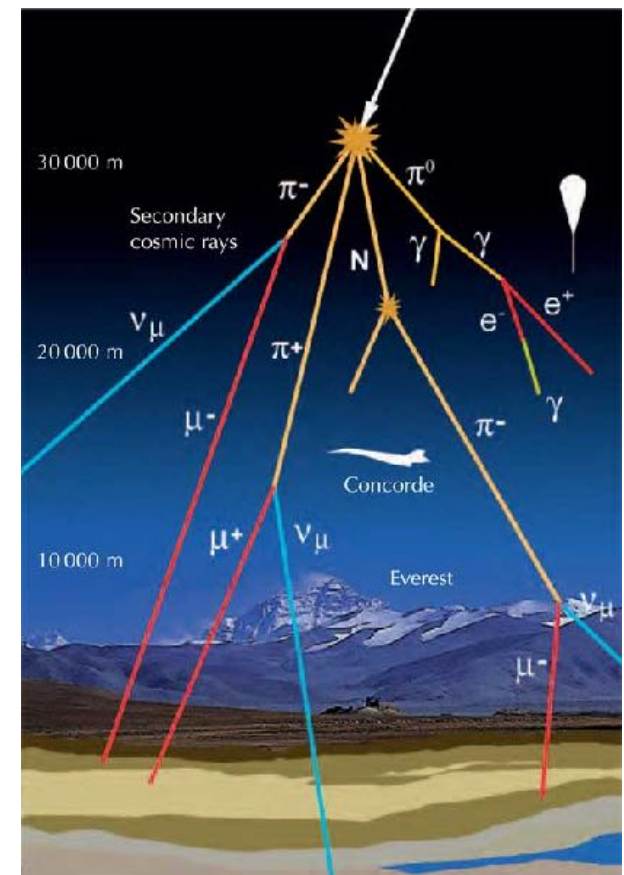
Length contraction

Let's move to the muon's frame. It “knows” it lives for 2.2microseconds, and it “knows” it can only travel 100s metres in that time.

How does it make it Earth?

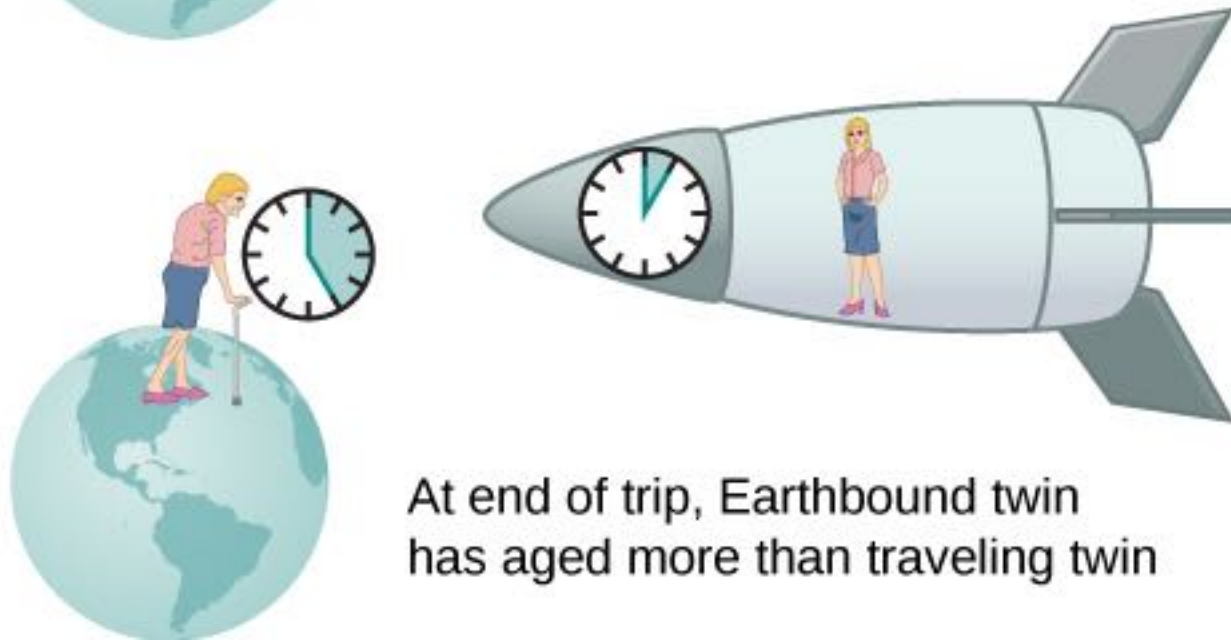
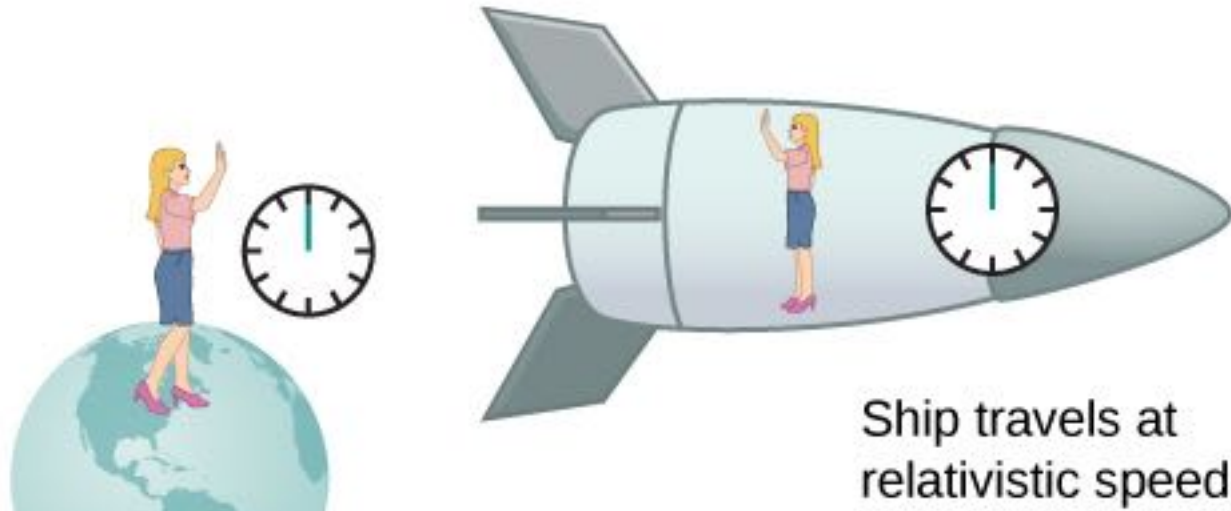
Earth is rushing towards the muon. Distance is “length contracted” from muon's frame

$$L_{\mu} = \frac{L_{\text{Earth}}}{\gamma}$$



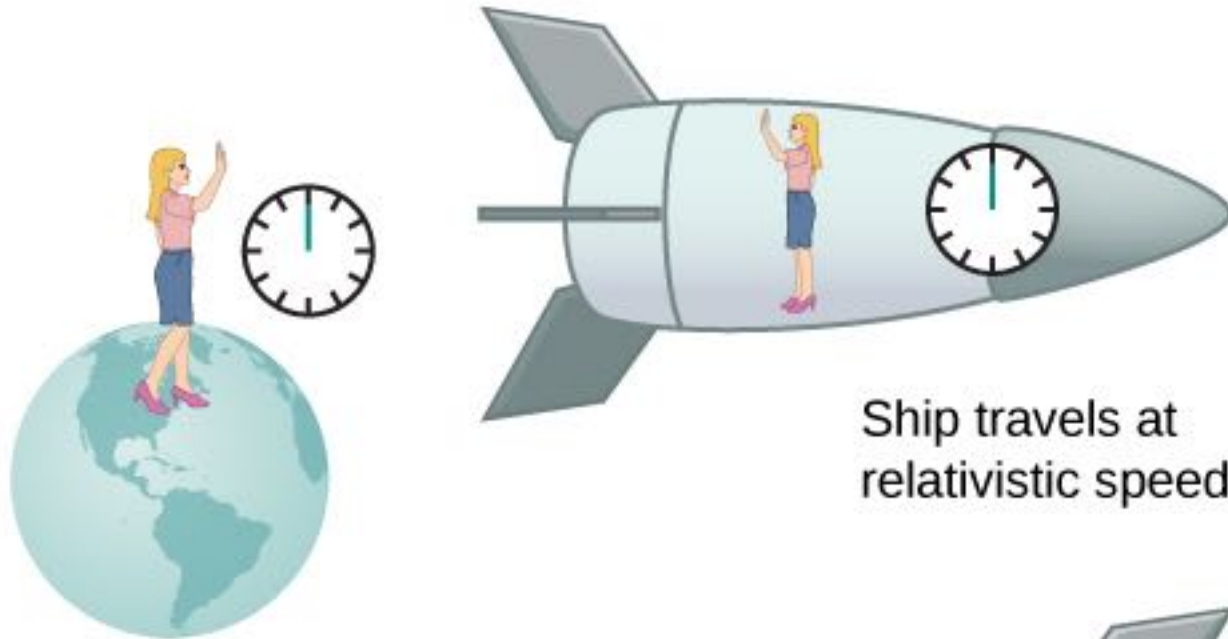
Twin Paradox

At start of trip, both twins are same age

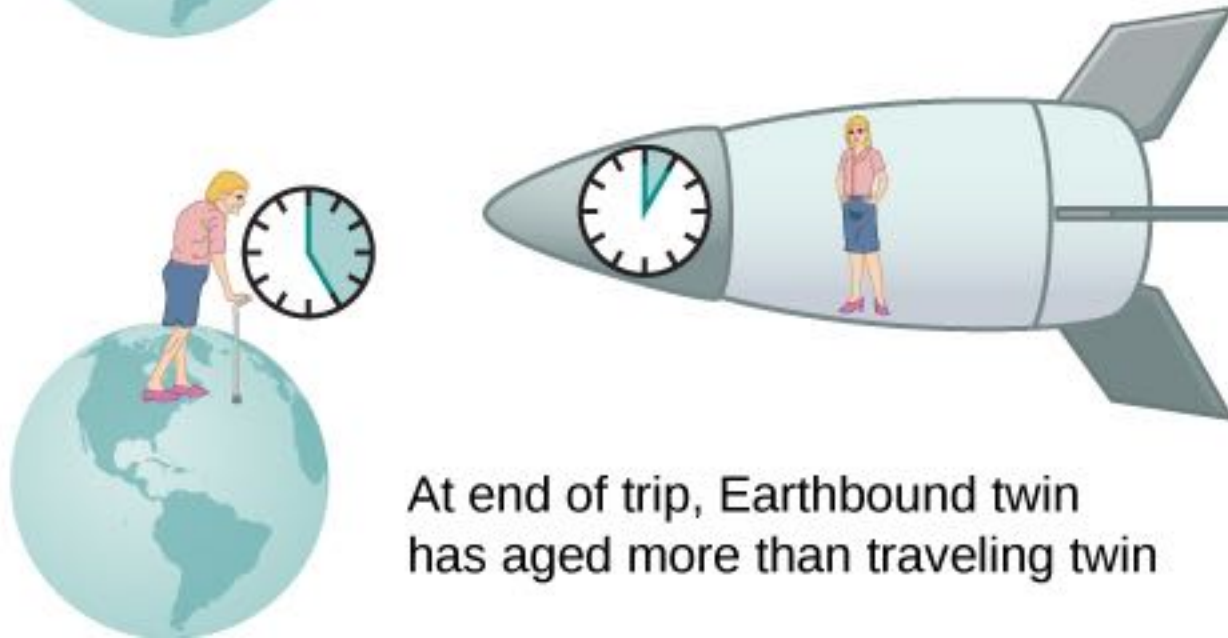


Twin Paradox

At start of trip, both twins are same age



Ship travels at relativistic speed

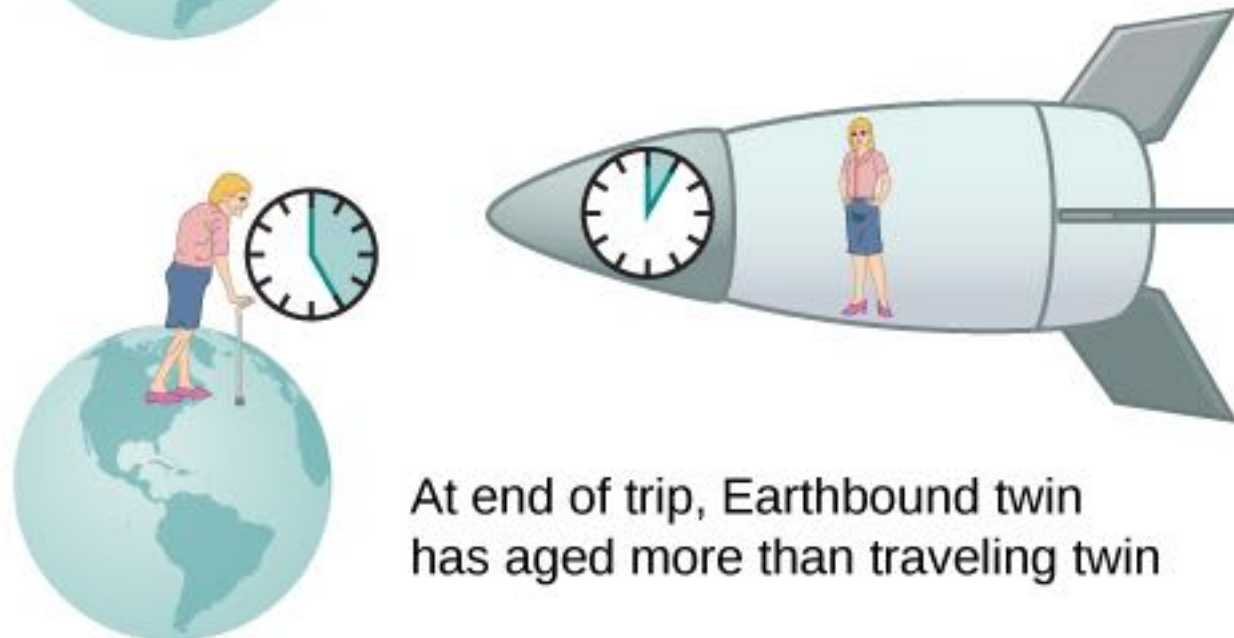
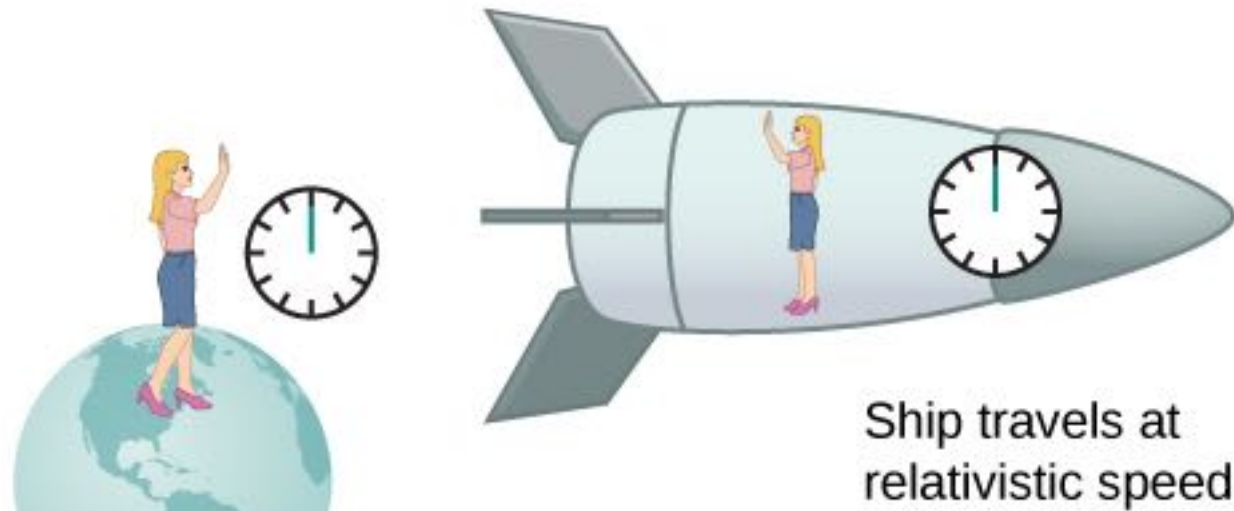


At end of trip, Earthbound twin has aged more than traveling twin

But if all motion is relative why can't I think about rocket staying still and Earth moving? Then Earth twin is younger!?

Twin Paradox

At start of trip, both twins are same age

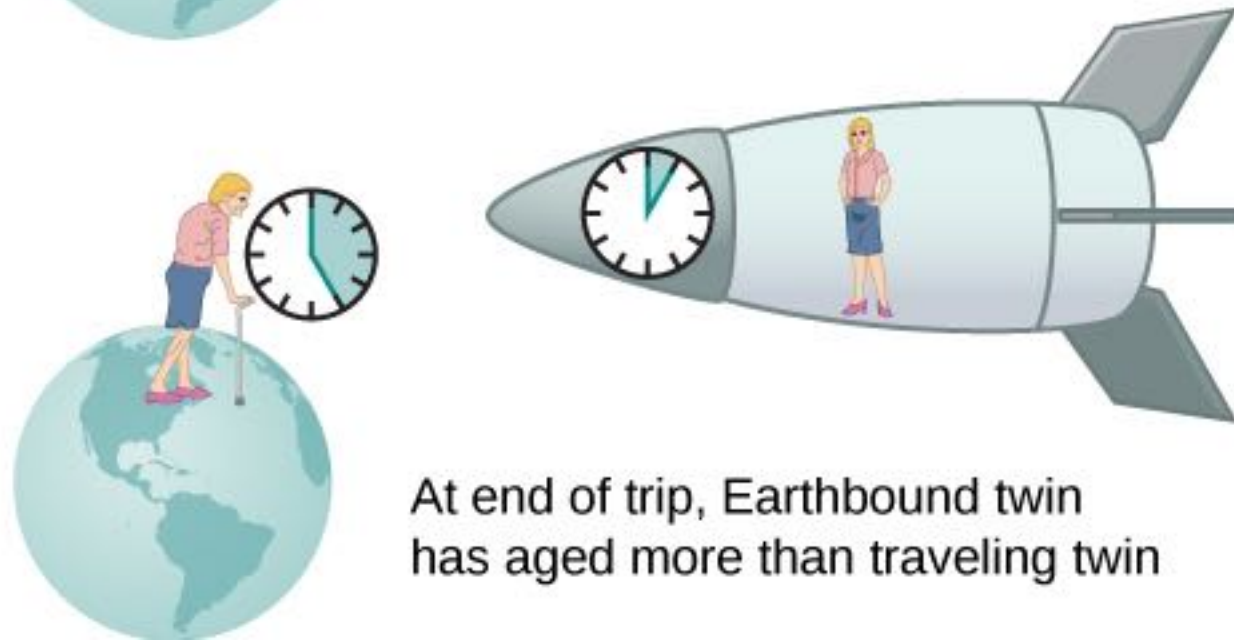
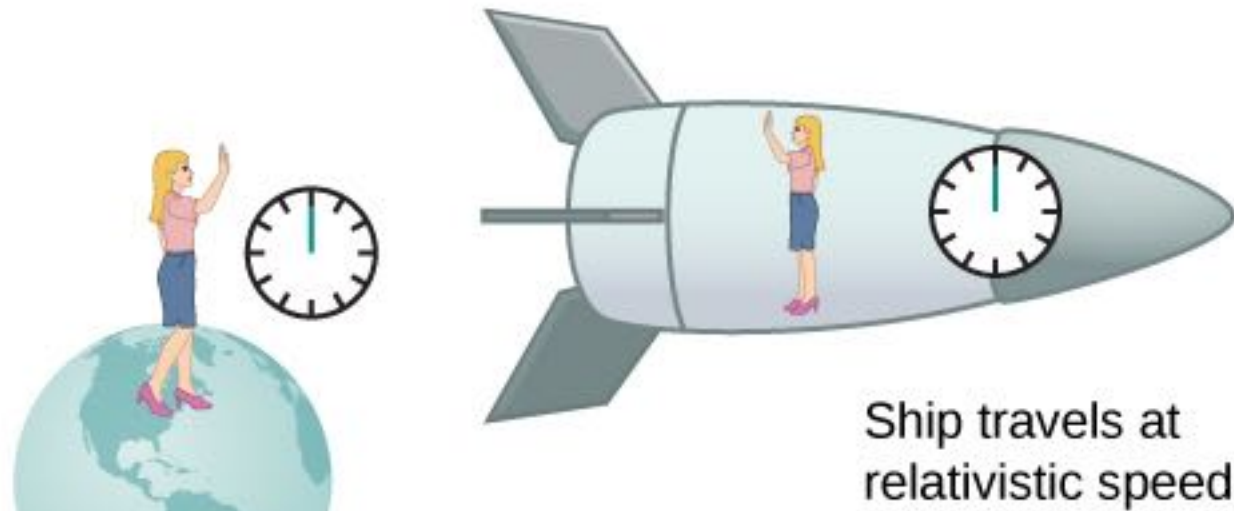


But if all motion is relative why can't I think about rocket staying still and Earth moving? Then Earth twin is younger!?

Only Earth twin stays in an inertial frame, the situation is not symmetric.

Twin Paradox

At start of trip, both twins are same age



But if all motion is relative why can't I think about rocket staying still and Earth moving? Then Earth twin is younger!?

Only Earth twin stays in an inertial frame, the situation is not symmetric.

Rocket twin will come back younger...relativity is weird!

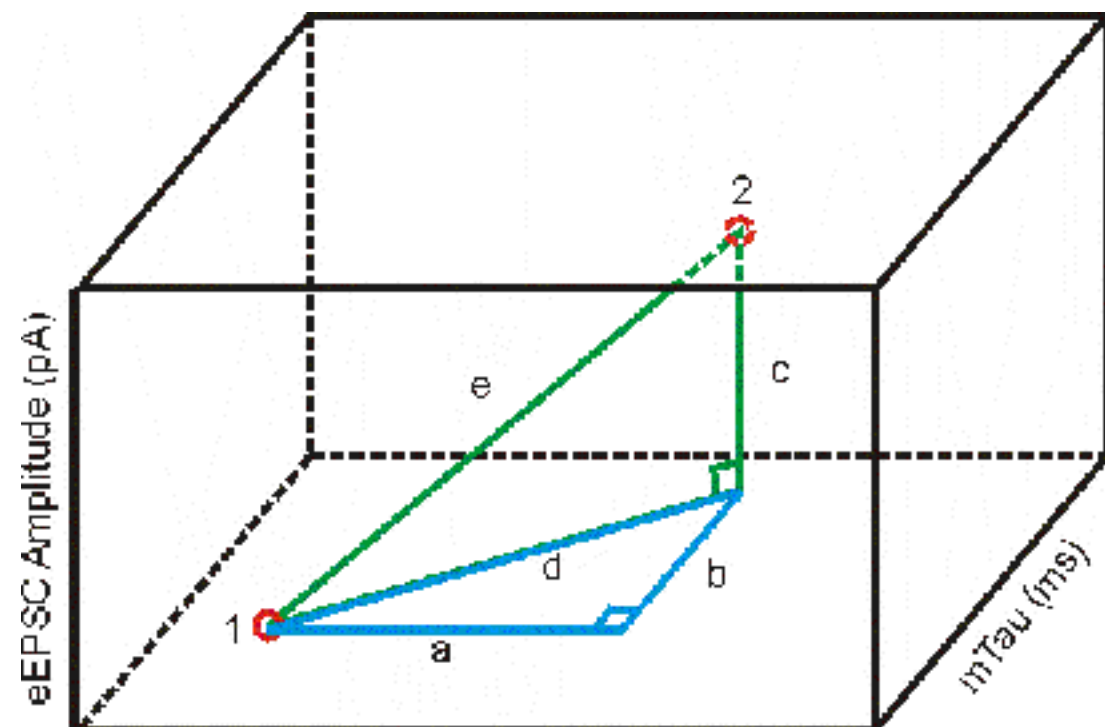
Spacetime

Classically there was absolute space and absolute time

In relativity all times and distances are relative AND time and space are not separate

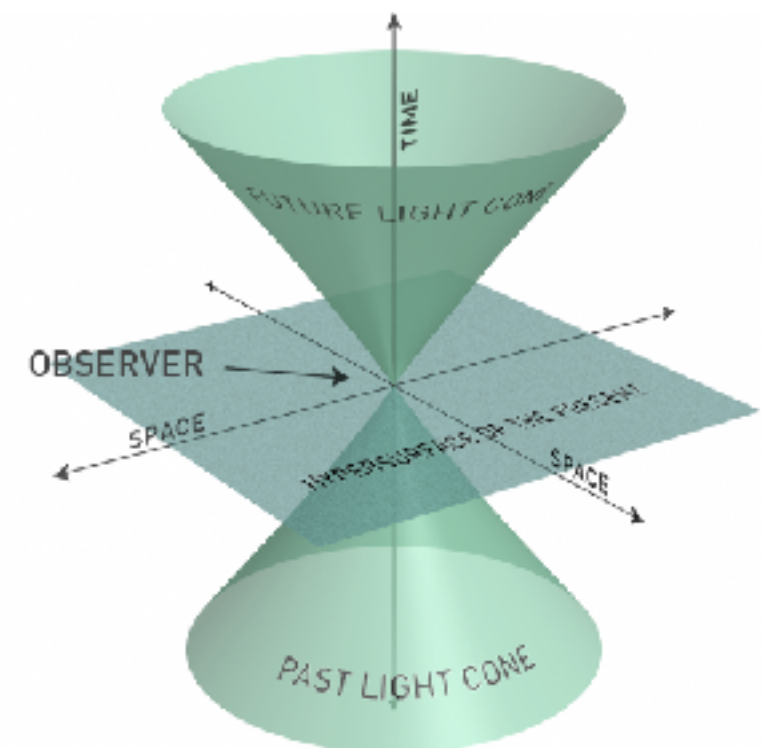
Distance

$$\Delta d^2 = \Delta x^2 + \Delta y^2 + \Delta z^2$$



Proper distance

$$\Delta s^2 = (c\Delta t)^2 - \Delta x^2 - \Delta y^2 - \Delta z^2$$



Relativistic

$$t' = \gamma t - \gamma \frac{v}{c^2} x$$

$$x' = \gamma x - \gamma v t$$

$$u = \frac{v + u'}{1 + \frac{vu'}{c^2}}$$

$$v \ll c$$



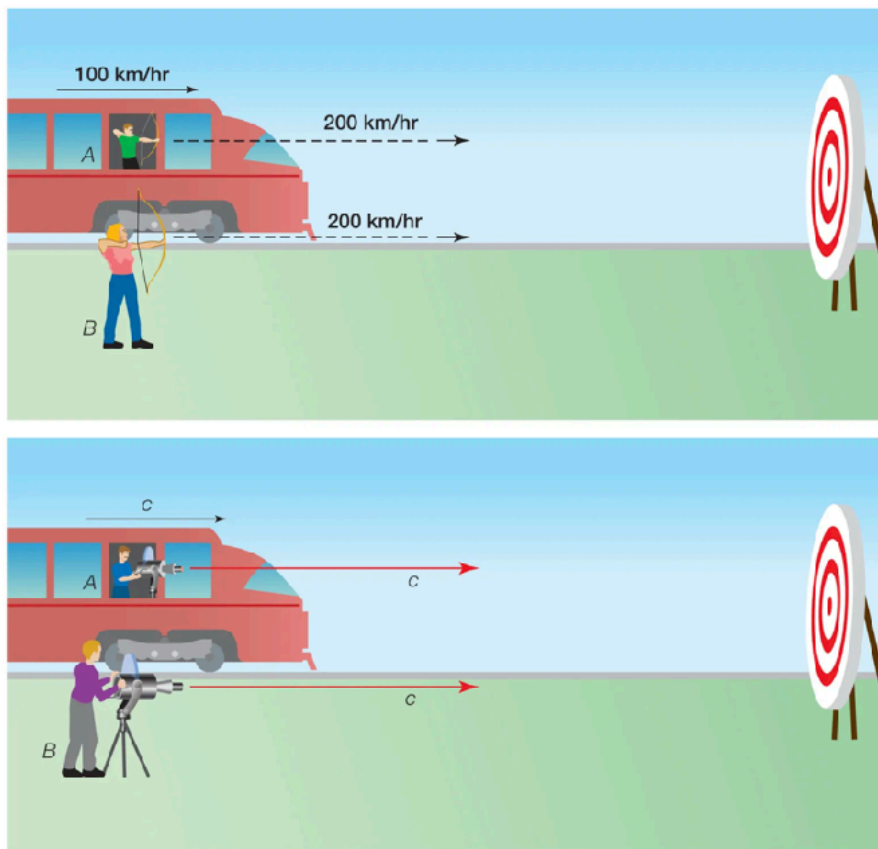
$$\gamma \rightarrow 1$$

Classical

$$t = t'$$

$$\vec{x}' = \vec{x} + \vec{v}t$$

$$u = v + u'$$

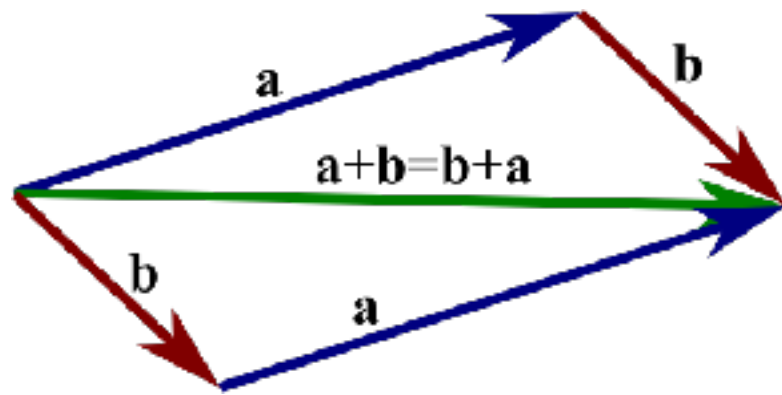


Nothing can travel faster than the speed of light

Spacetime — four vectors

Distance

$$\Delta d^2 = \Delta x^2 + \Delta y^2 + \Delta z^2$$



$$\vec{x} = \begin{pmatrix} x \\ y \\ z \end{pmatrix}$$

$$|\vec{x}| = \sqrt{x^2 + y^2 + z^2}$$

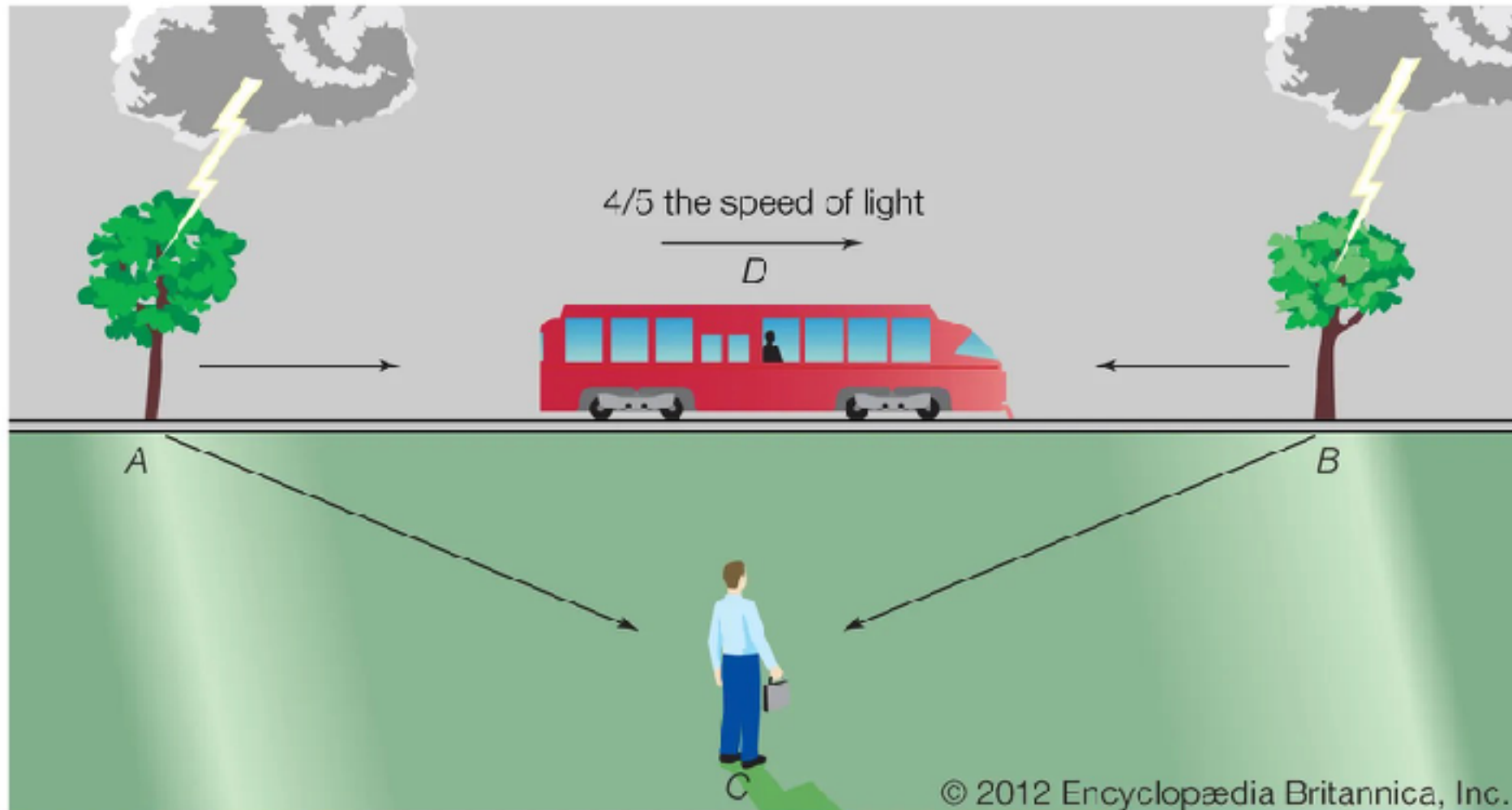
Proper distance

$$\Delta s^2 = (c\Delta t)^2 - \Delta x^2 - \Delta y^2 - \Delta z^2$$

$$x^\mu = \begin{pmatrix} ct \\ x \\ y \\ z \end{pmatrix}$$

$$|x^\mu|^2 = (ct)^2 - x^2 - y^2 - z^2$$

Simultaneity



Spacetime separation the same for both observers, but one sees them simultaneously, the other sees B first

$$\Delta s^2 = (c\Delta t)^2 - \Delta x^2 - \Delta y^2 - \Delta z^2$$

Spacetime — four vectors

Momentum

$$\vec{p} = \begin{pmatrix} p_x \\ p_y \\ p_z \end{pmatrix} = m\vec{v}$$

Four momentum

$$p^\mu = \begin{pmatrix} E/c \\ p_x \\ p_y \\ p_z \end{pmatrix}$$

$$|p^\mu|^2 = \left(\frac{E}{c}\right)^2 - \vec{p}^2$$

Spacetime — four vectors

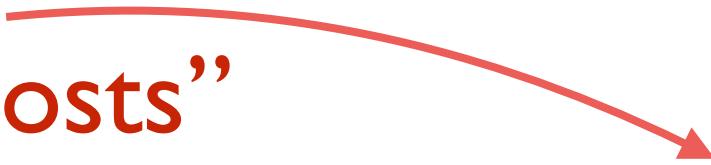
Momentum

$$\vec{p} = \begin{pmatrix} p_x \\ p_y \\ p_z \end{pmatrix} = m\vec{v}$$

Four momentum

$$p^\mu = \begin{pmatrix} E/c \\ p_x \\ p_y \\ p_z \end{pmatrix}$$

Invariant under “boosts”


$$|p^\mu|^2 = \left(\frac{E}{c}\right)^2 - \vec{p}^2$$

Spacetime — four vectors

Momentum

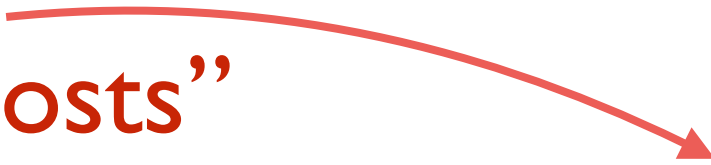
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Invariant under “boosts”

Always equal to $m^2 c^2$


$$|p^\mu|^2 = \left(\frac{E}{c}\right)^2 - \vec{p}^2$$

Spacetime — four vectors

Momentum

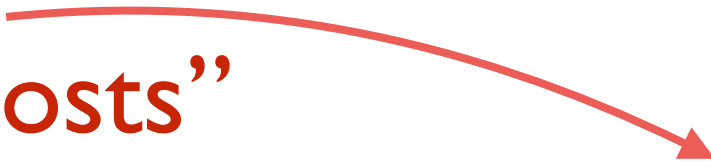
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Invariant under “boosts”

Always equal to $m^2 c^2$


$$|p^\mu|^2 = \left(\frac{E}{c}\right)^2 - \vec{p}^2$$

$$E^2 = m^2 c^4 + p^2 c^2$$

Most Famous Physics formula of all time?

$$E^2 = m^2 c^4 + p^2 c^2$$


Stationary particles

$$E = mc^2$$

Massless particles (light)

$$E = pc$$

$$E=mc^2$$

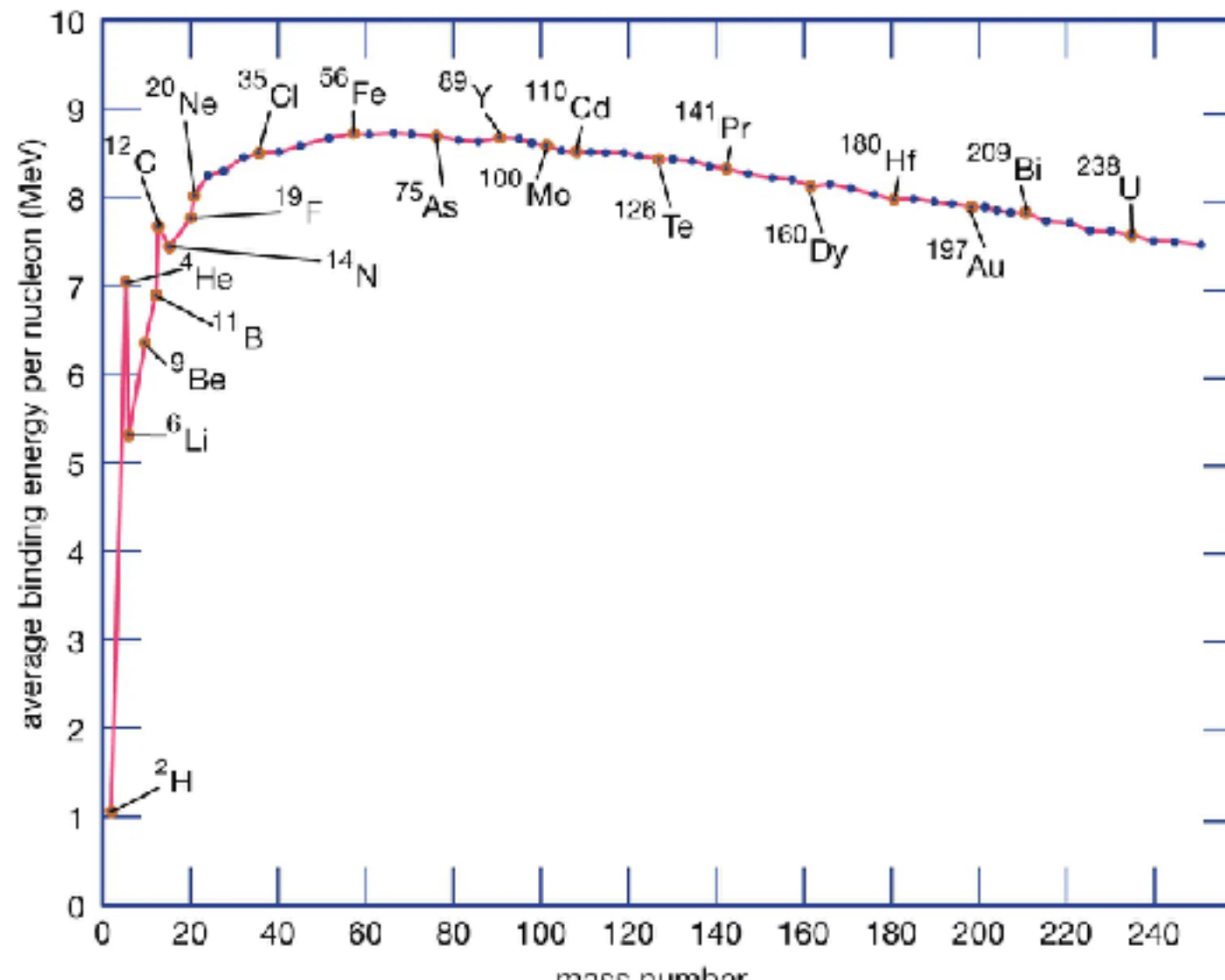
$$E = mc^2$$

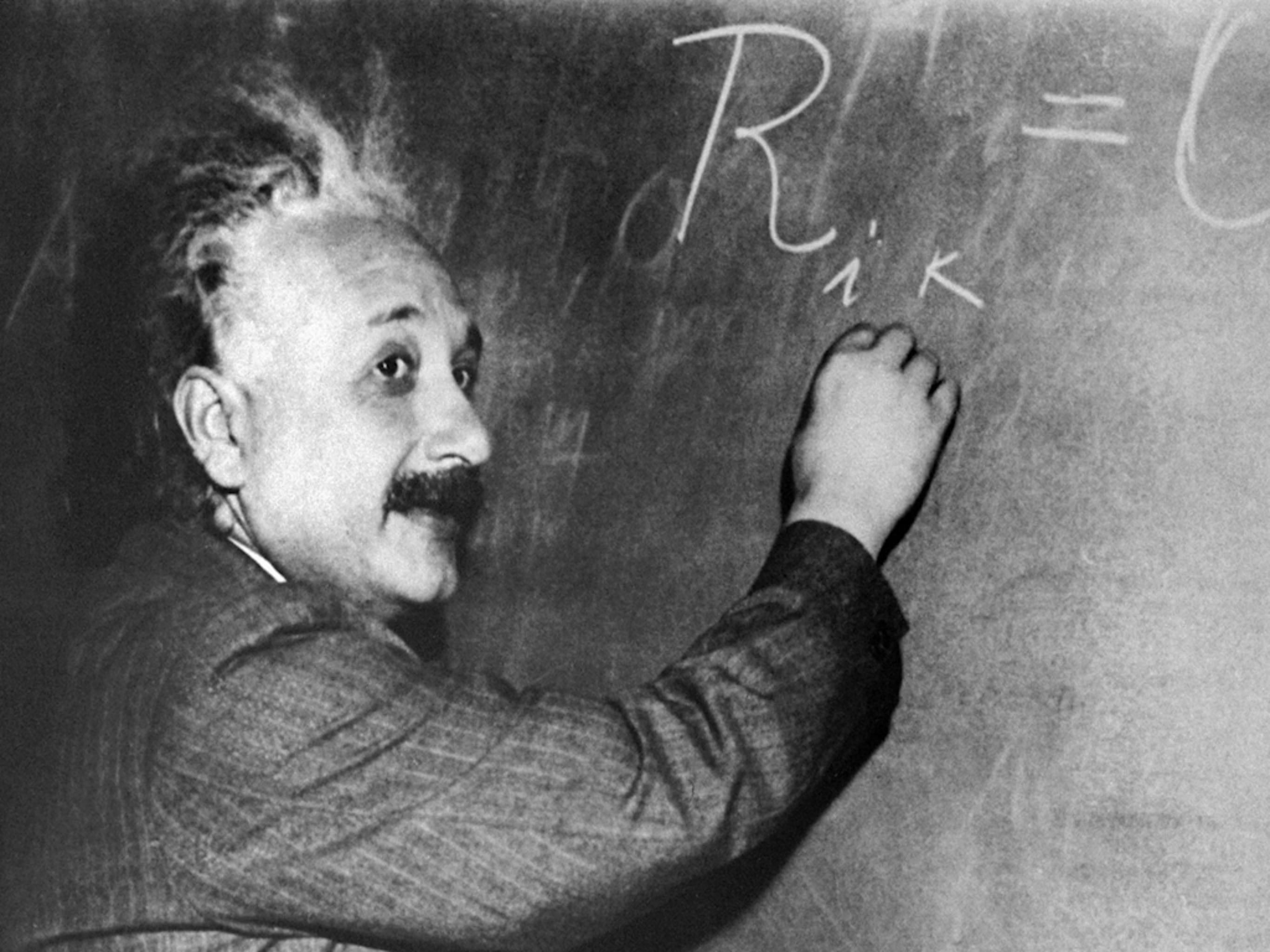
Turn mass into energy

c is a big number

1kg results in 9E16 Joule

All US energy usage in a year is 1E20 Joule

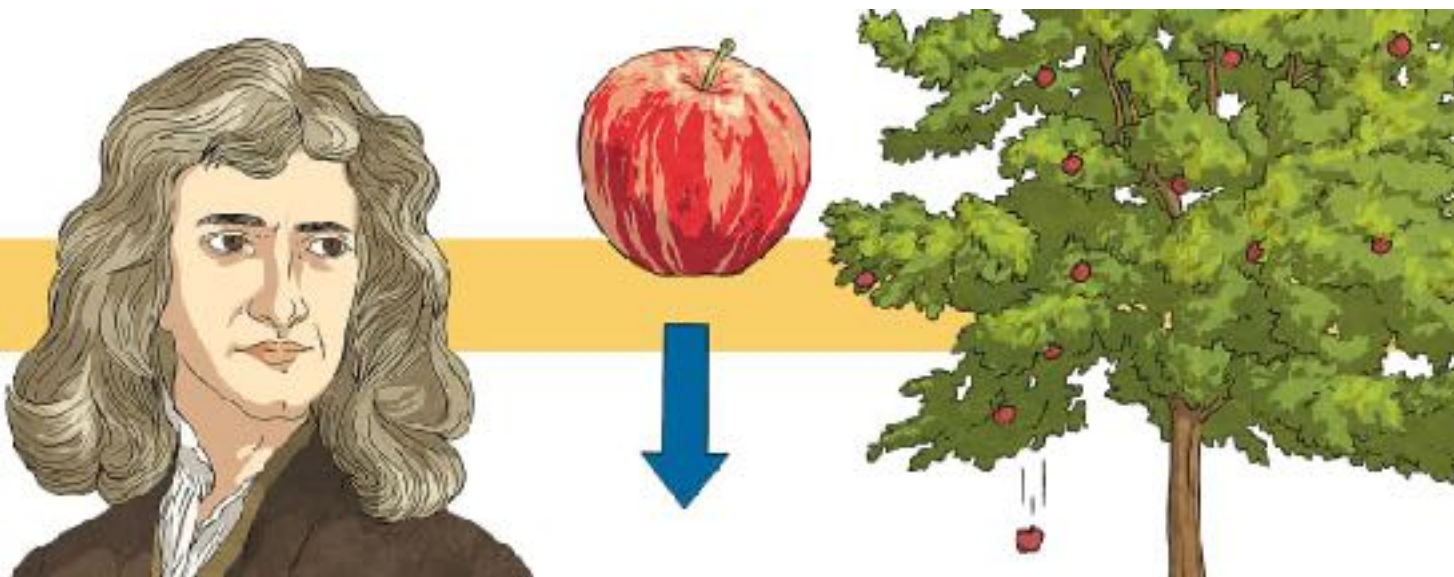




$$R_{ik} = 0$$

General Relativity

How to deal with acceleration, non-inertial frames?
Took Einstein 10 years, but in 1915 he published his
General Theory...a tour de force in physics.
A relativistic theory of gravity.



Newtonian Gravity

$$F = G_N \frac{m_1 m_2}{r^2}$$

Does not obey rules of special relativity!

Principle of Equivalence

$$F = ma$$

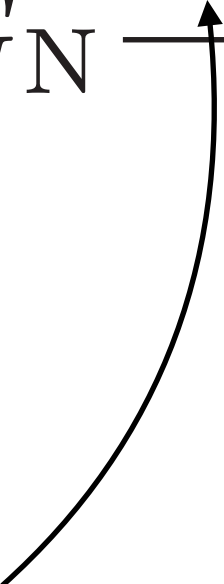
$$F = G_{\text{N}} \frac{m_1 m_2}{r^2}$$

Principle of Equivalence

$$F = ma$$

$$F = G_N \frac{m_1 m_2}{r^2}$$

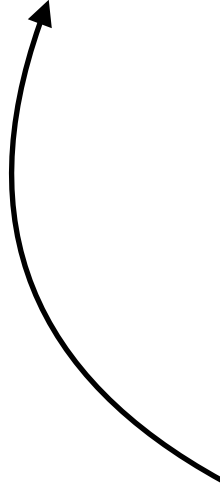
Gravitational mass



Principle of Equivalence

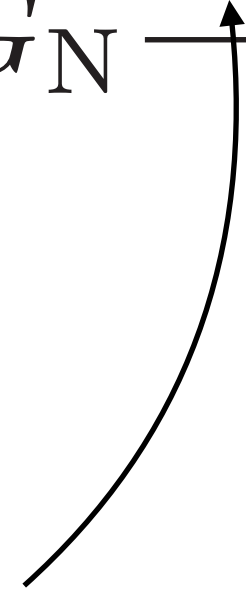
$$F = ma$$

Inertial mass



$$F = G_{\text{N}} \frac{m_1 m_2}{r^2}$$

Gravitational mass



Principle of Equivalence

$$F = ma$$

Inertial mass

$$F = G_N \frac{m_1 m_2}{r^2}$$

Gravitational mass

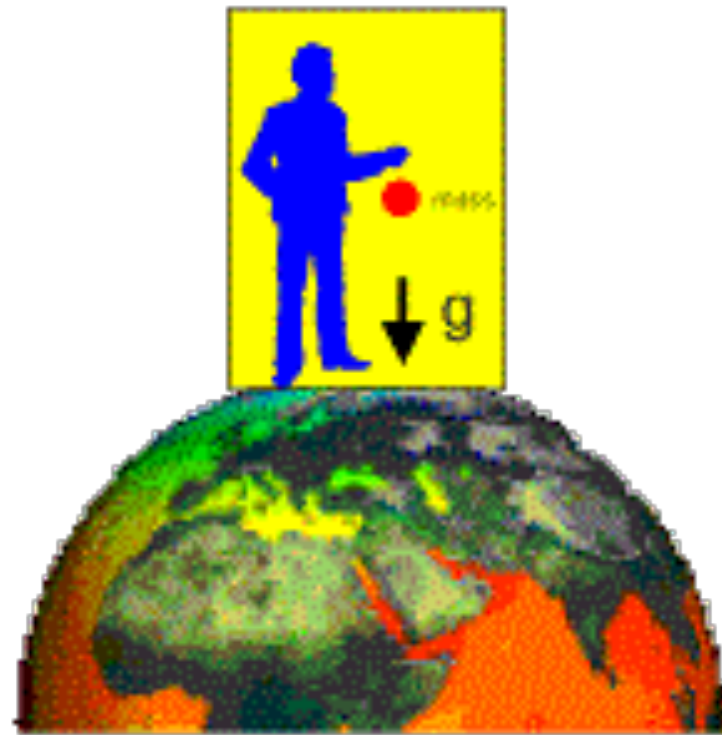


Principle of Equivalence

$$F = ma$$

Inertia

$$g = a$$



gravitational
mass



inertial
mass

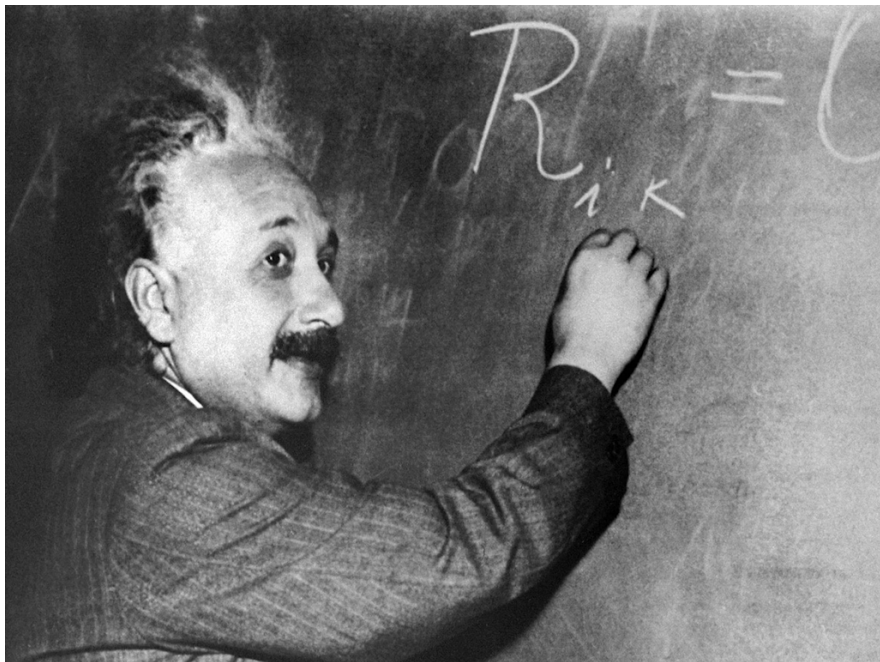
$$F_N = \frac{m_1 m_2}{r^2}$$

gravitational mass

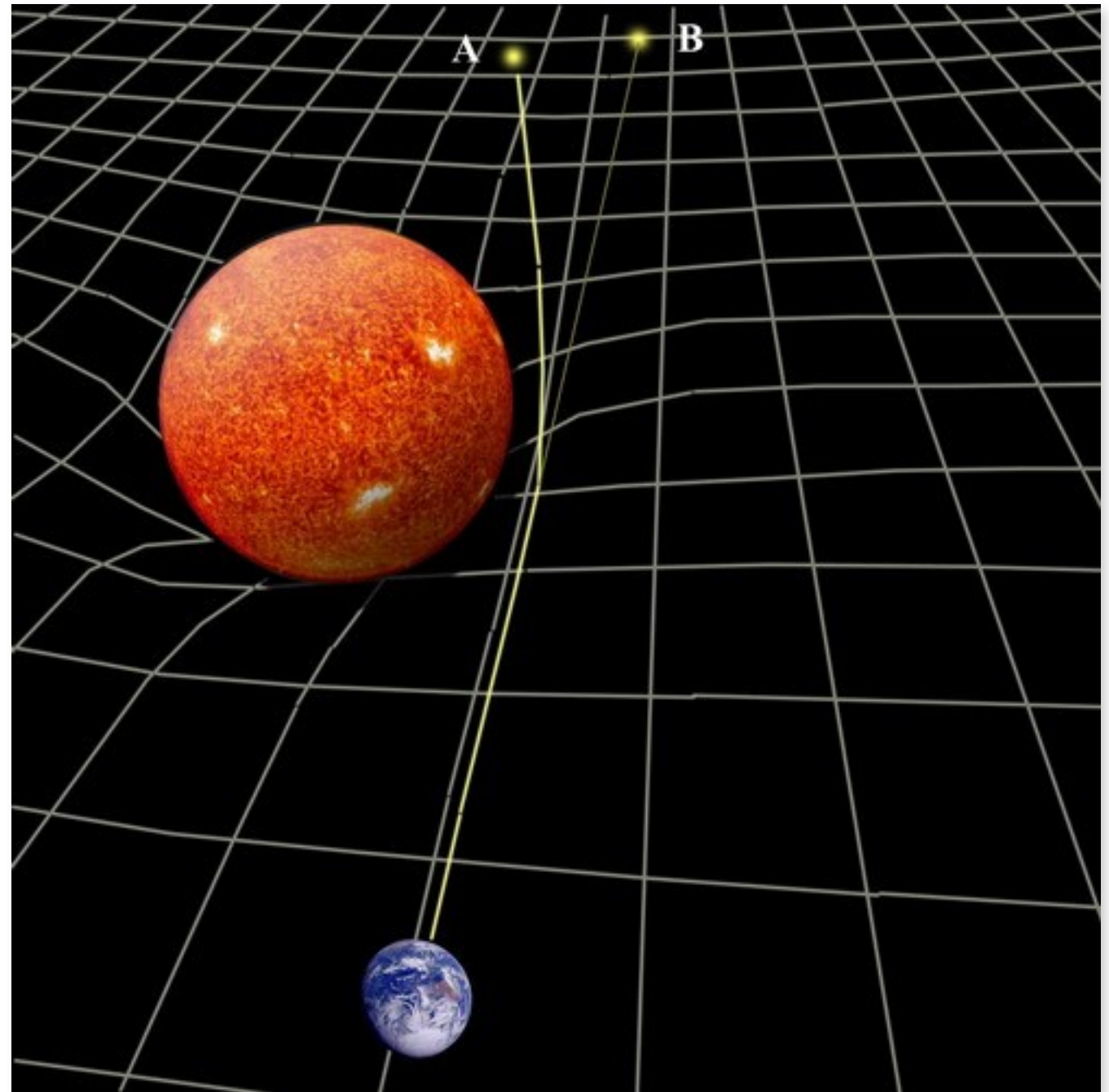
Universality of free fall

General Relativity

Freely falling particles move along “geodesics” in the curved spacetime, responding to mass/energy



$$\underbrace{R_{\mu\nu}}_{\text{curvature}} - \frac{1}{2}g_{\mu\nu}\underbrace{R}_{\text{mass/energy}} = \frac{8\pi G}{c^4}T_{\mu\nu}$$

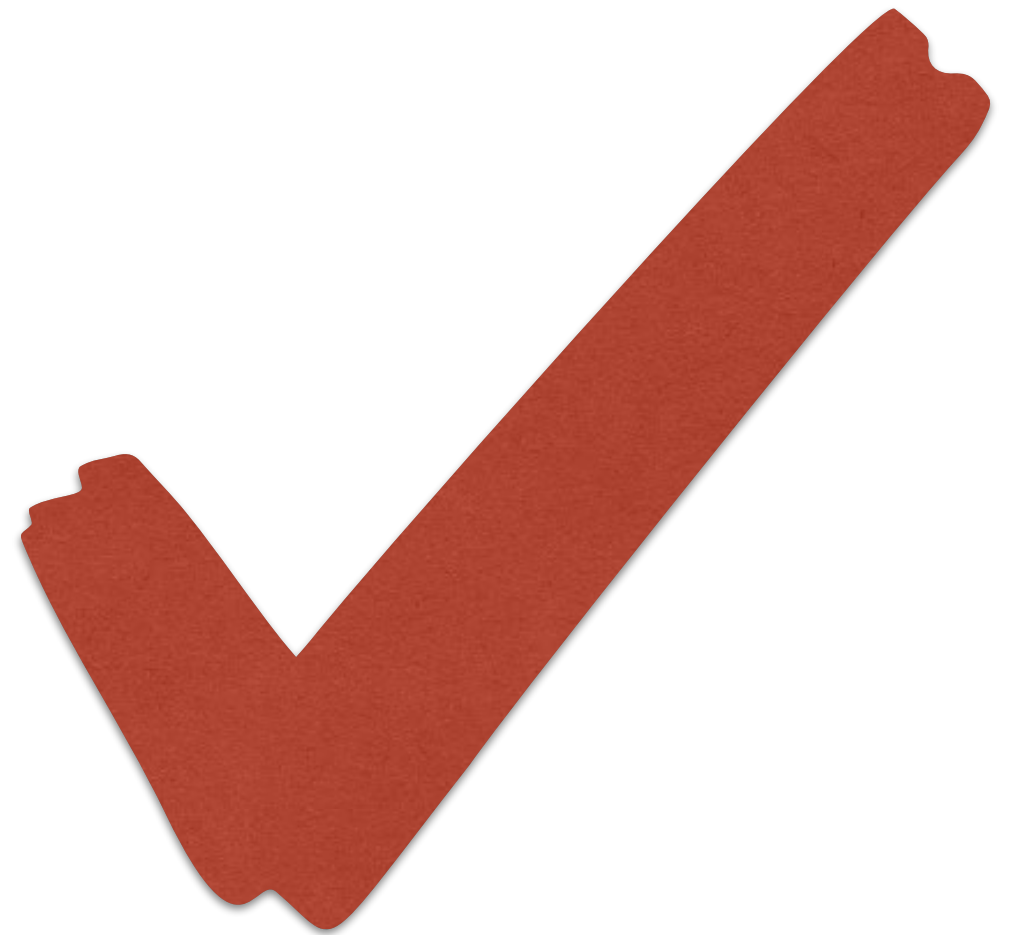


Predictions

- Perihelion advance of Mercury
- Gravitational waves
- Black Holes
- Strong lensing
- Gravitational redshift
- ...

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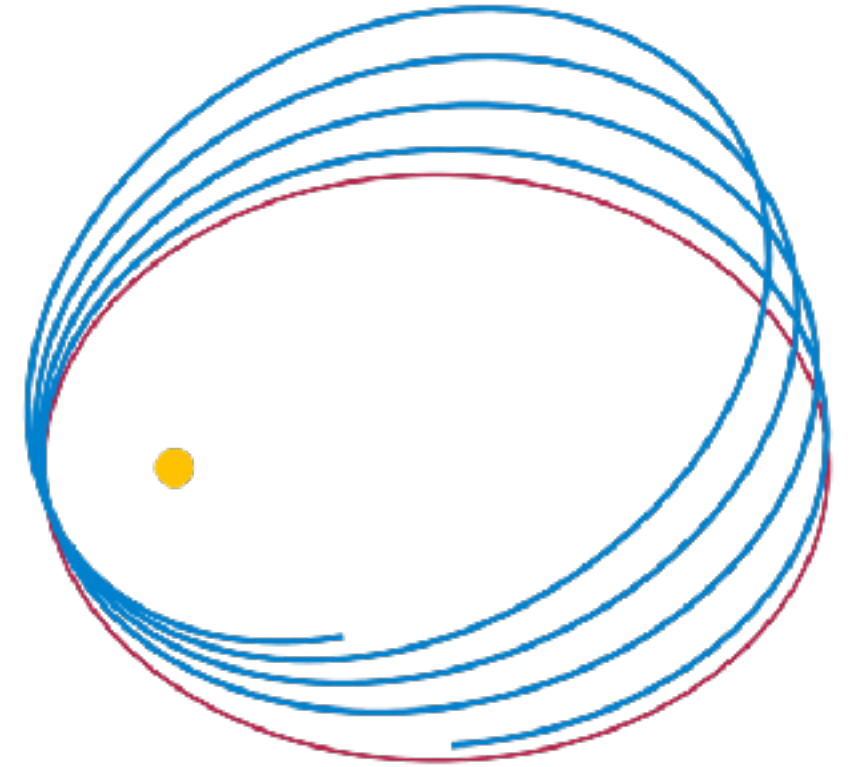


Perihelion advance of Mercury

Closest approach of Mercury to Sun changes with time

Not predicted to happen in Newtonian gravity (closed elliptical orbits)

Explained unaccounted for observed effect



Urbain Le Verrier (1859)

Bending of light

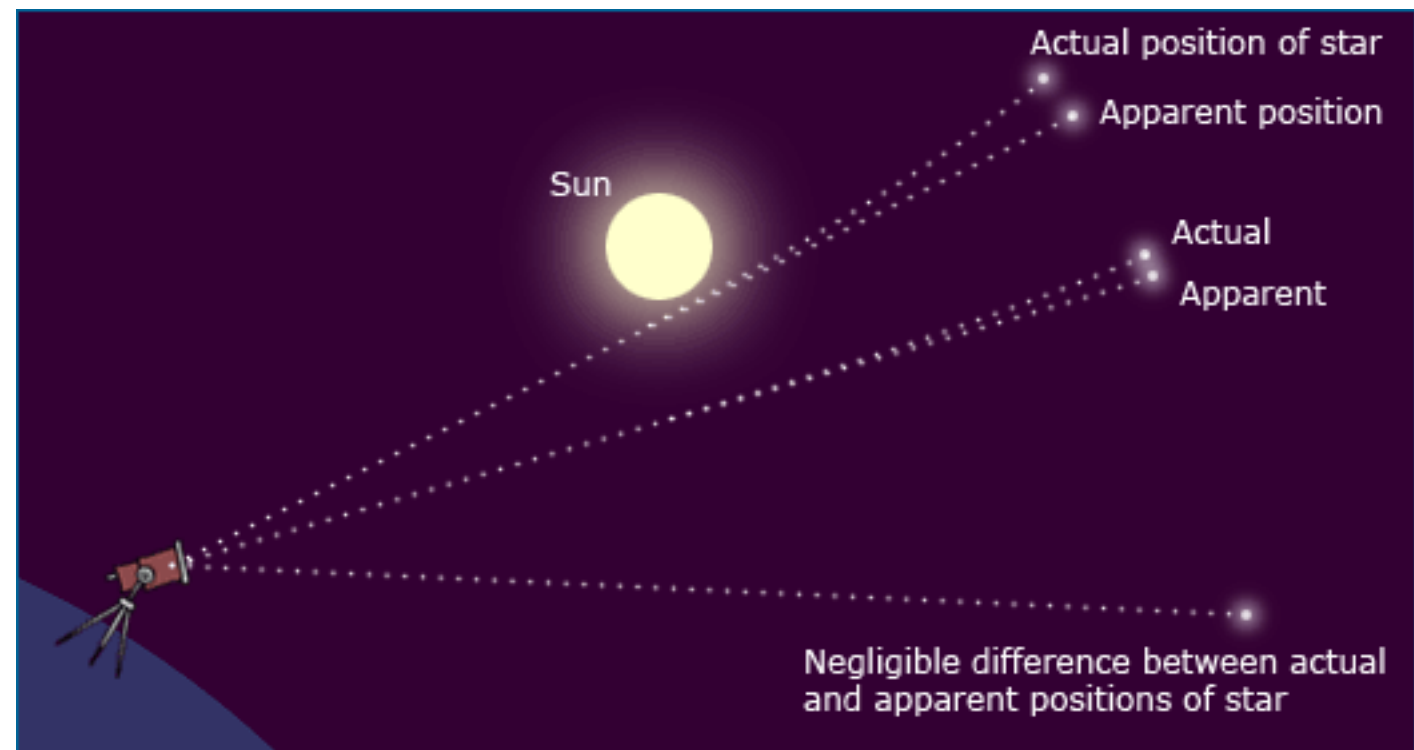
Light follows “null” geodesics, and bends when close to heavy object.

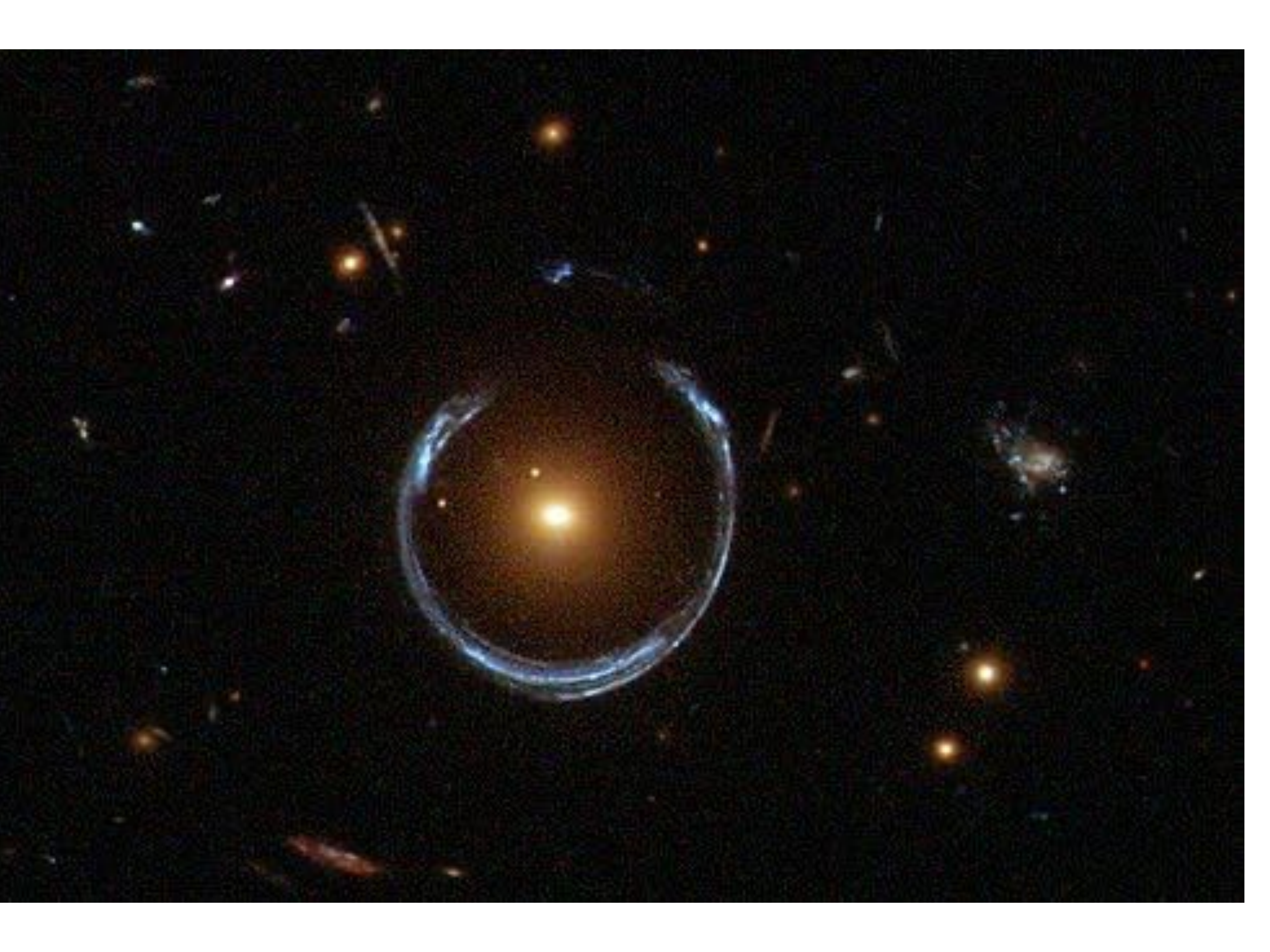
Effect is small (but GR prediction is twice Newtonian)

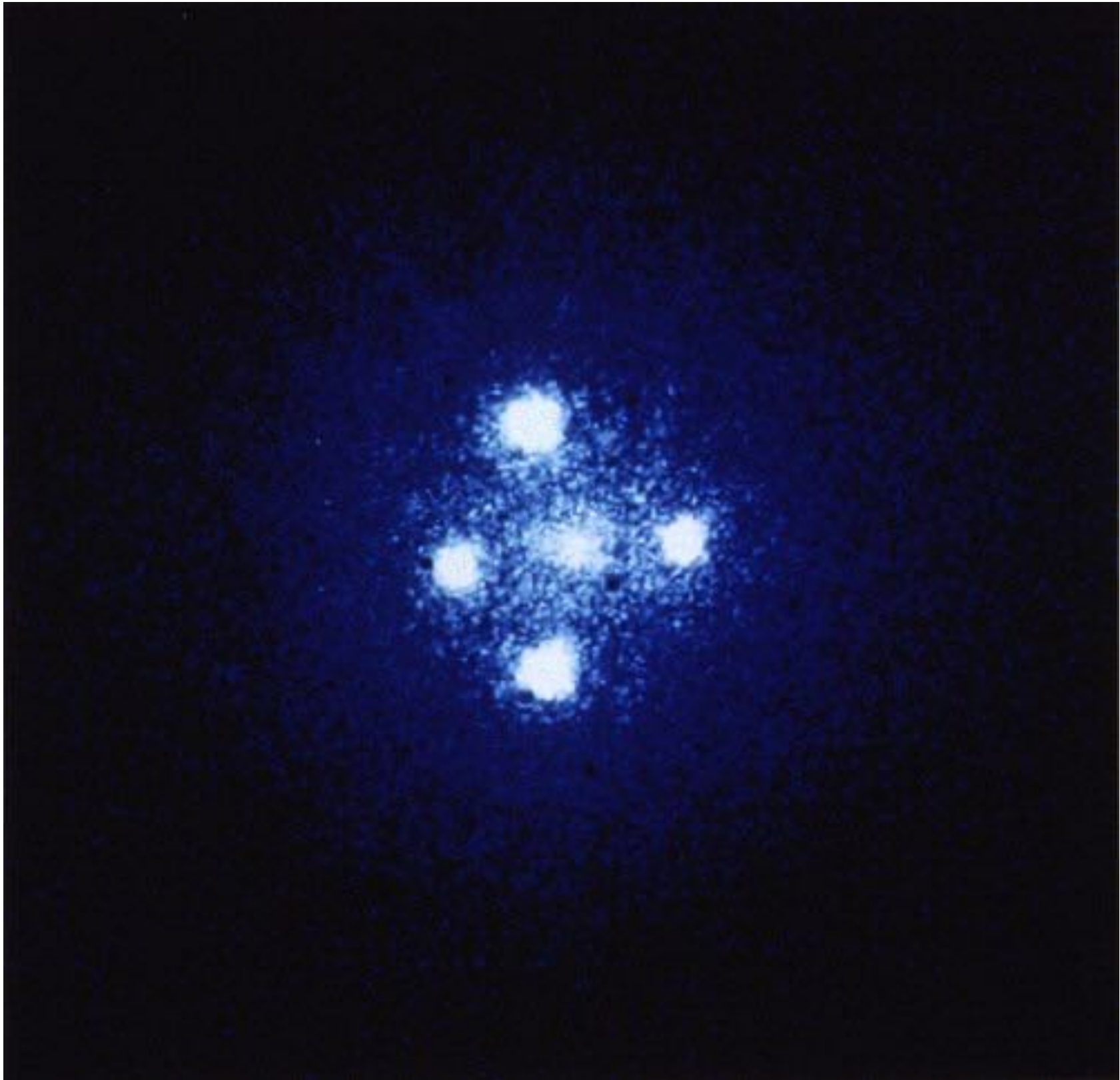
Eddington observed this effect in 1919



Solar eclipse, 1919

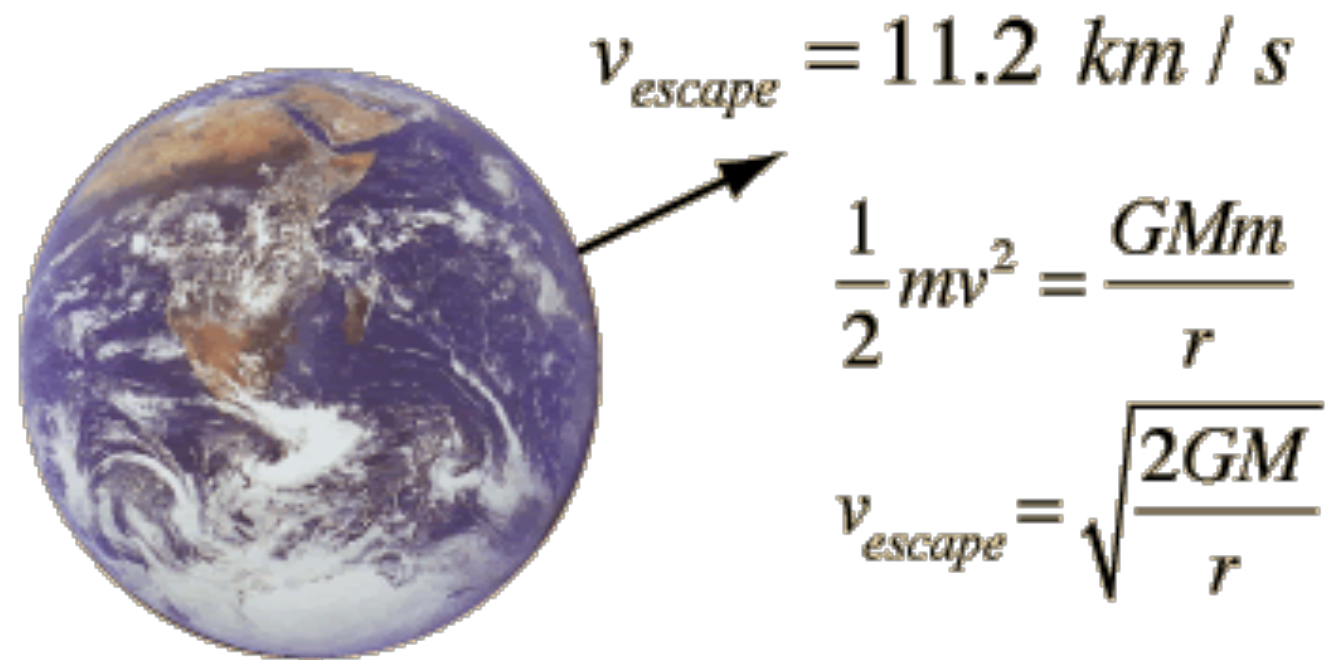




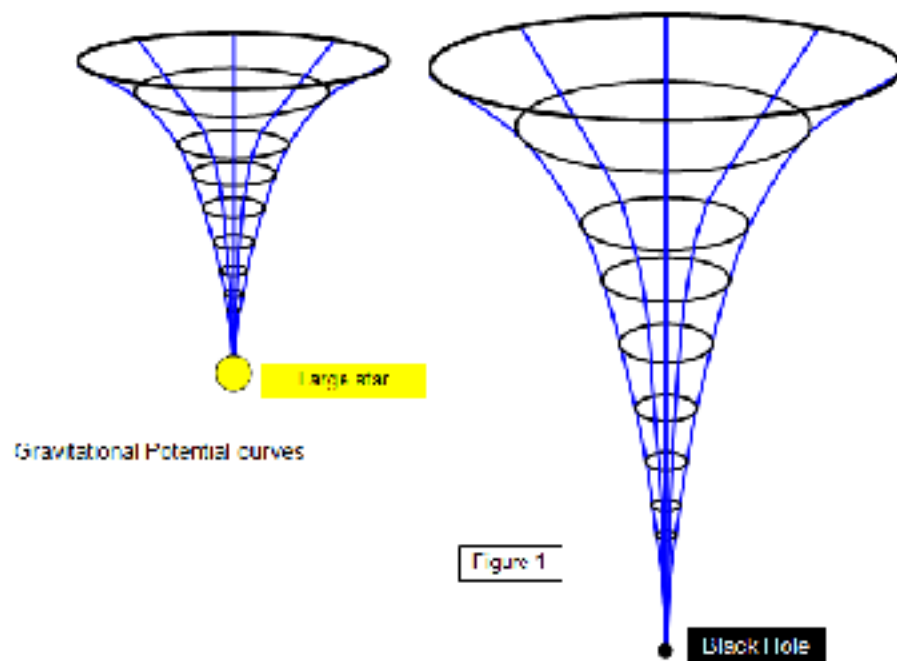


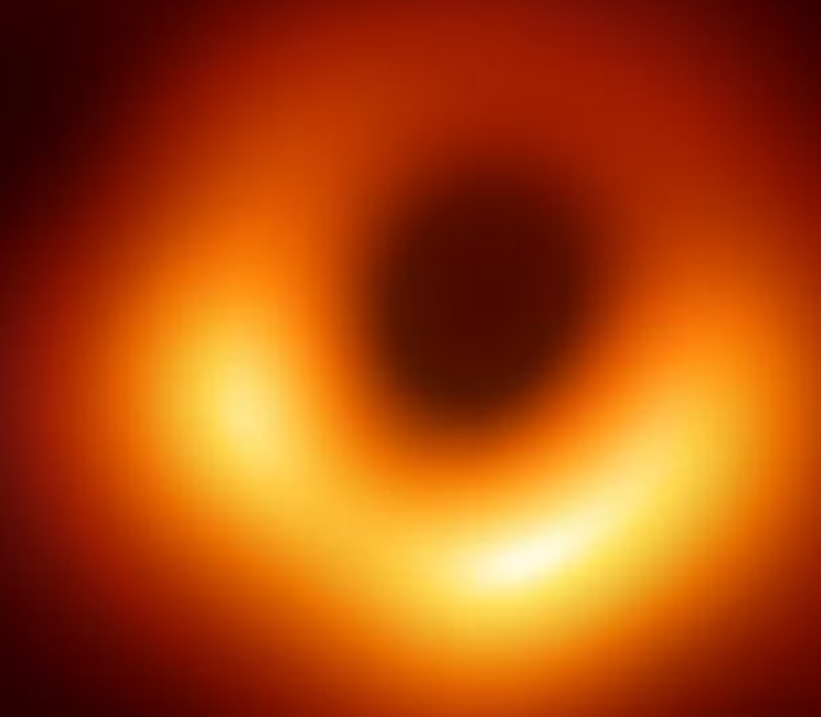


Black Holes



With enough mass in a small enough space escape velocity can reach the speed of light producing a **black hole**



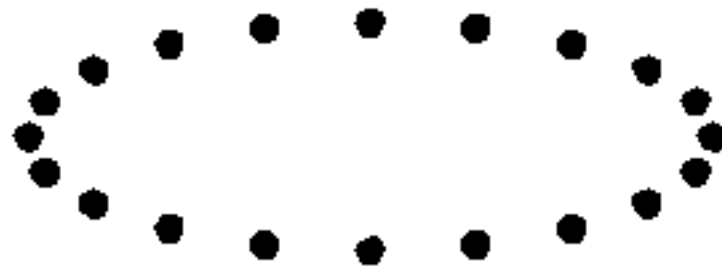


Andrea Ghez,
2020



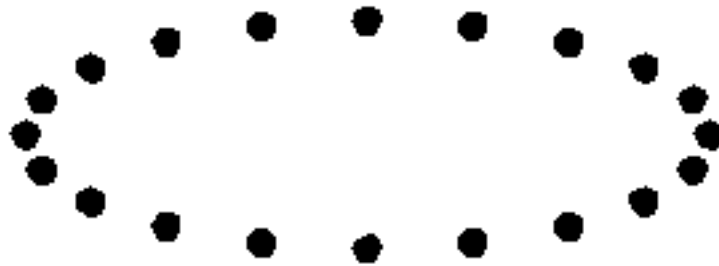
Gravitational Waves

Like shaking charges produce light, shaking mass can produce propagating gravitational waves



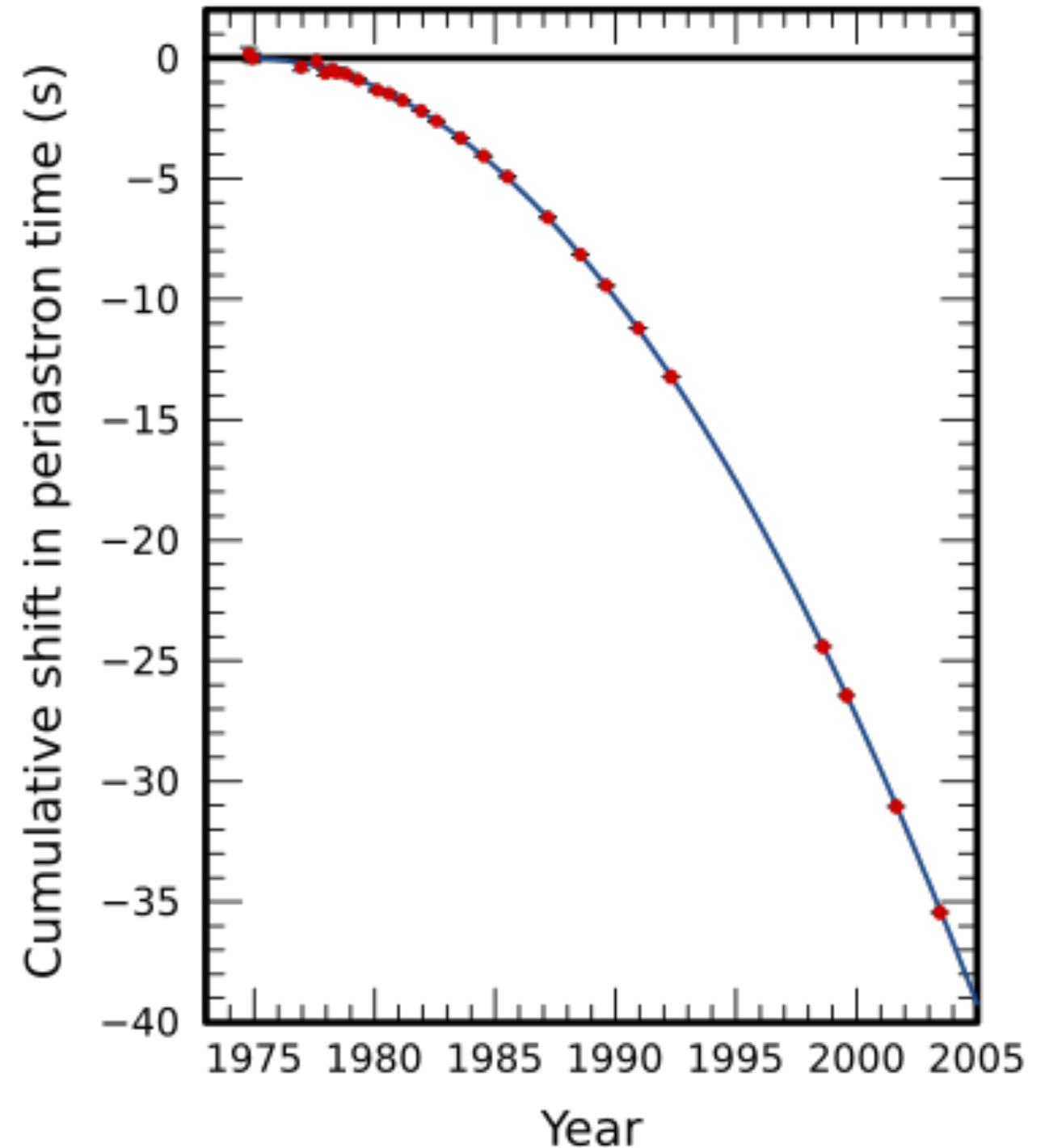
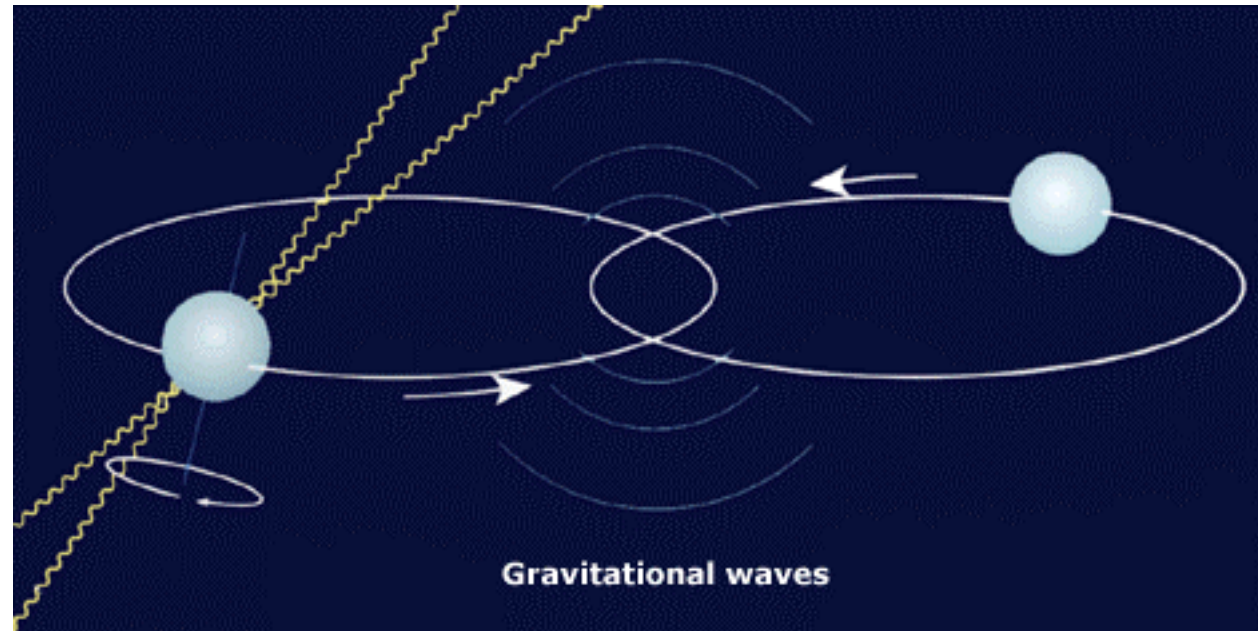
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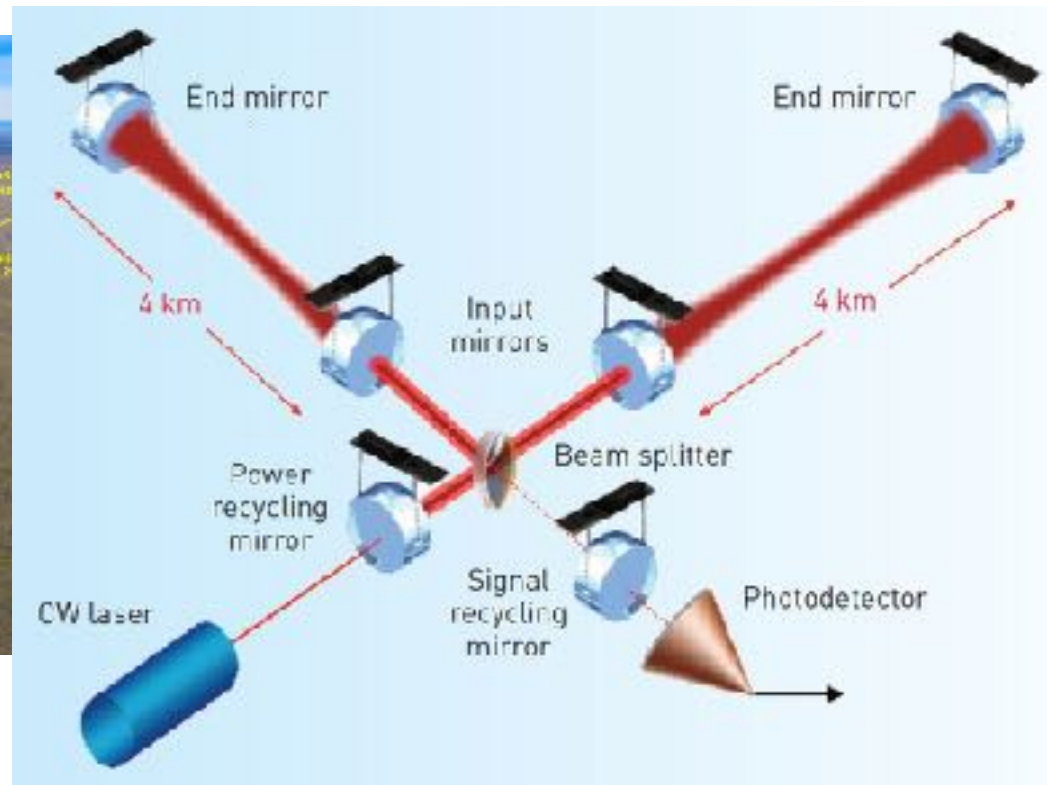


Hulse-Taylor Binary Pulsar

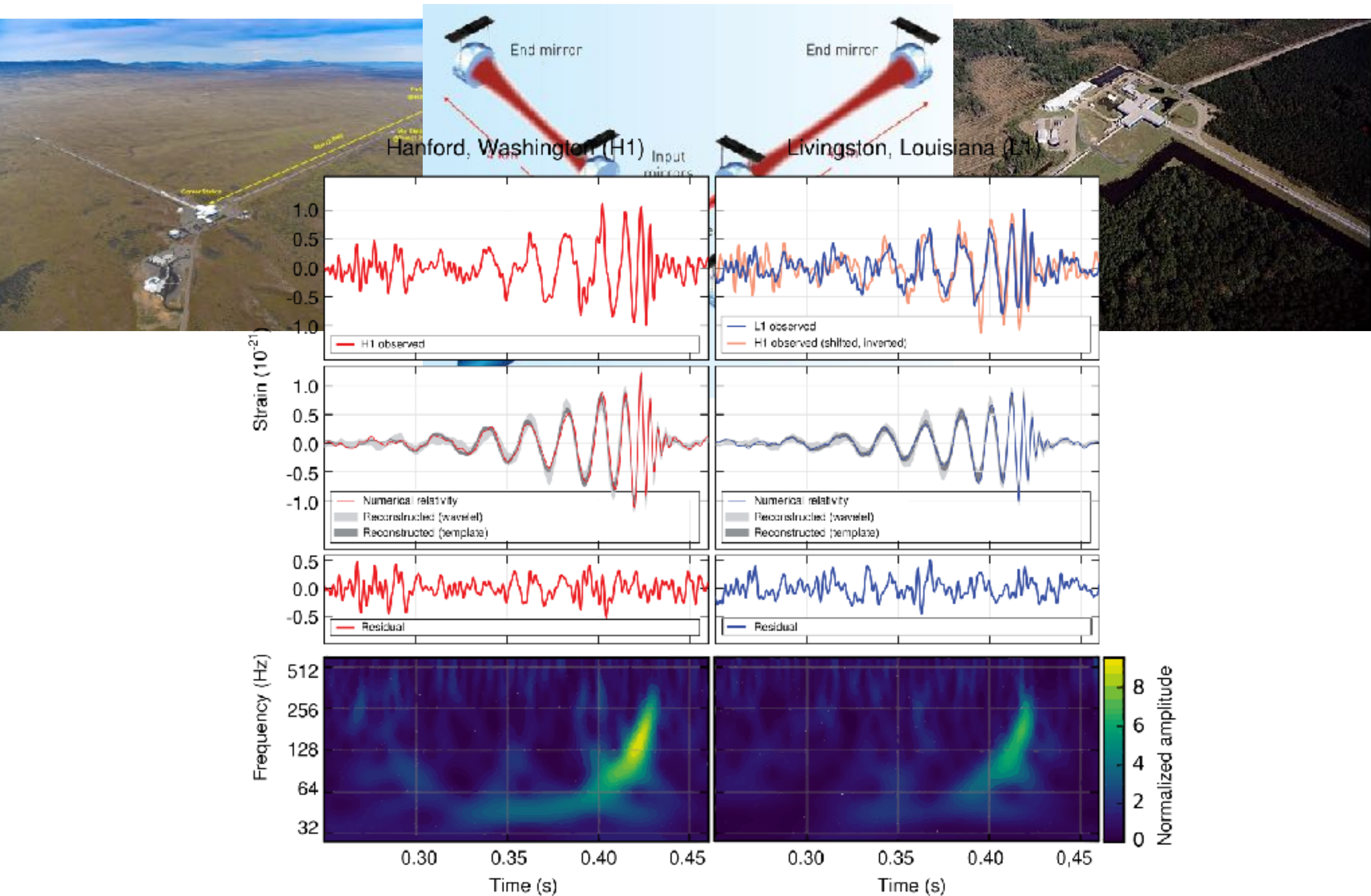
Indirect observation of gravitational waves



LIGO-Laser Interferometer Gravitational-wave Observatory

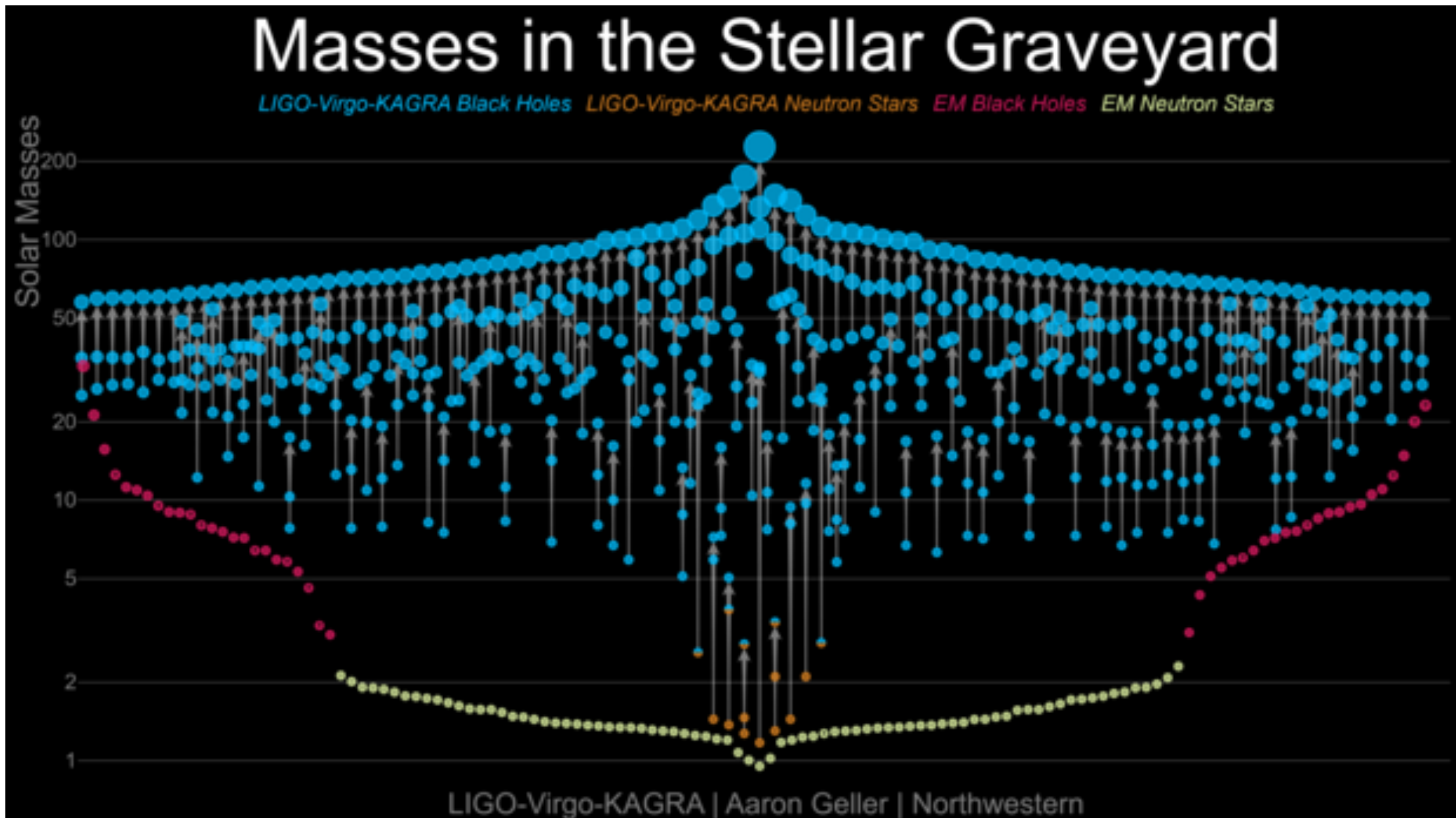


LIGO-Laser Interferometer Gravitational-wave Observatory



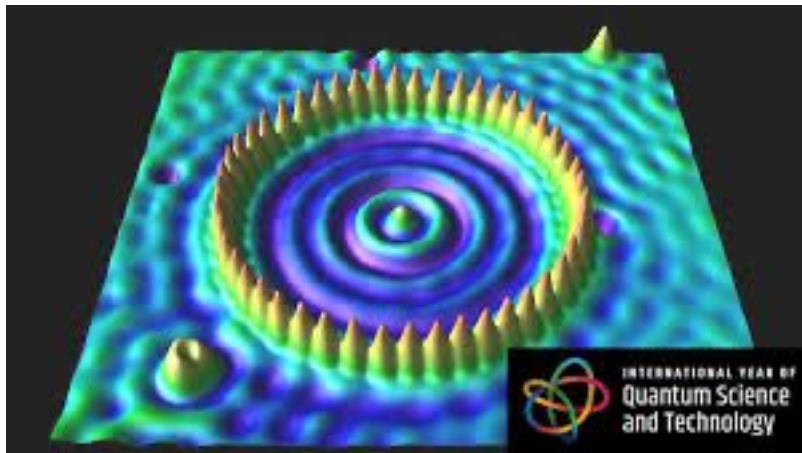
LIGO-Laser Interferometer Gravitational-wave Observatory

A new way to “listen” to the universe

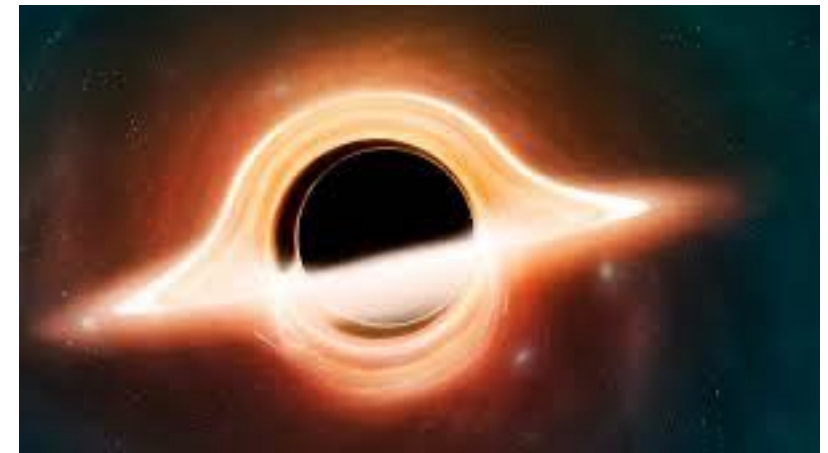


Relativity into the future

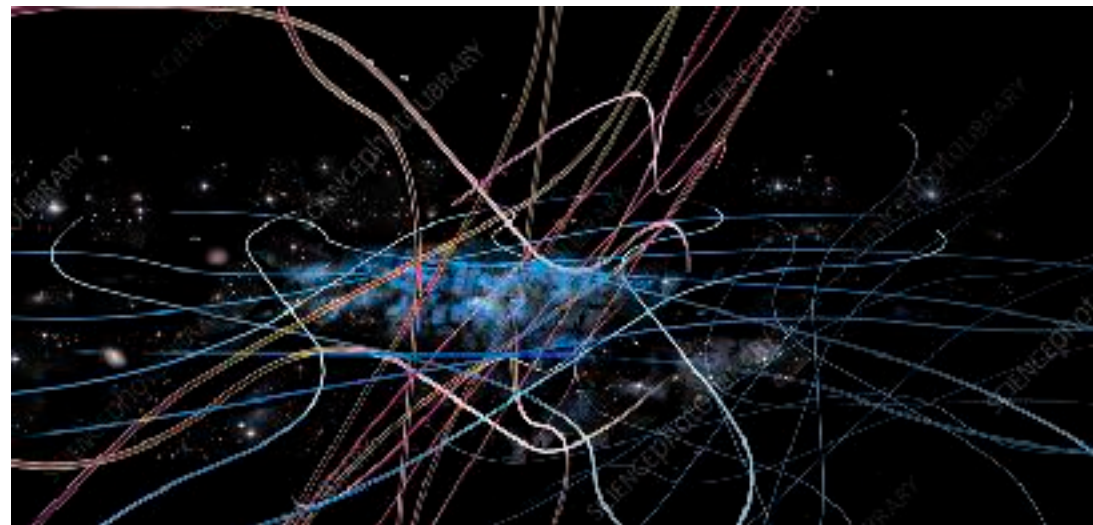
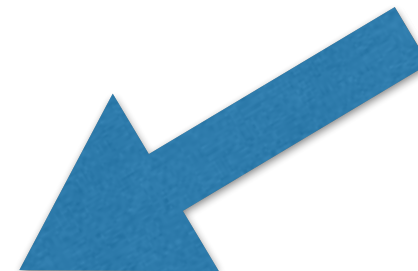
Relativity predicts its own downfall-spacetime singularities



Quantum Mechanics



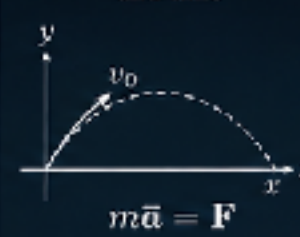
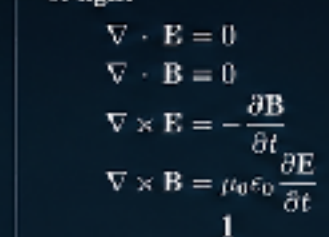
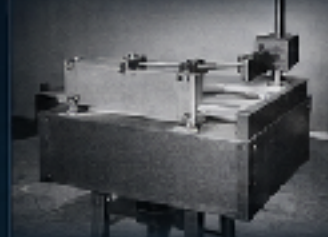
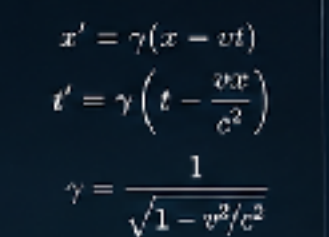
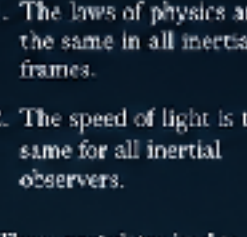
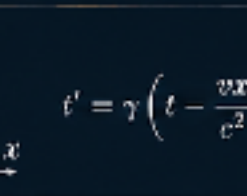
General Relativity/Gravity



String Theory/Quantum Gravity

The development of relativity

From Galileo's principle of relativity to Einstein's spacetime

1632 Galileo Galilei	1687 Isaac Newton	1860s James Clerk Maxwell	1887 Michelson–Morley	1890s–1905 Lorentz and Poincaré	1905 Albert Einstein	1908 Hermann Minkowski	1907–1915 Einstein
							
Principle of relativity for mechanics	Newtonian mechanics and absolute space and time	Electromagnetism and a fixed speed of light	Search for the ether	Lorentz transformations and a relativity principle	Special relativity	Spacetime geometry	General relativity
							
Experiments performed inside a uniformly moving ship cannot reveal the ship's constant velocity.	Formulated mechanics with absolute time and space. Newton's laws are invariant under Galilean transformations, so all inertial frames are dynamically equivalent.	Maxwell's equations predict electromagnetic waves with a fixed speed c , seemingly incompatible with Galilean relativity.	Null result: no evidence for Earth's motion through a luminiferous ether. Sharpened the conflict between Maxwell's theory and Galilean relativity.	Lorentz found the transformations that leave Maxwell's equations invariant. Poincaré recognized their group structure and developed a relativity principle.	These postulates lead to the Lorentz transformations, relativity of simultaneity, time dilation, length contraction, and $E = mc^2$.	Reformulated special relativity as four-dimensional spacetime with invariant interval $ds^2 = -c^2 dt^2 + dx^2 + dy^2 + dz^2$.	The equivalence principle: gravity and acceleration are locally indistinguishable. Gravity is spacetime curvature: $G_{\mu\nu} = 8\pi G T_{\mu\nu}$.
Galilean relativity				Special relativity		General relativity	
Inertial frames are equivalent ($t' = t, x' = x - vt$)				Lorentz symmetry, constant speed of light		Spacetime curvature	
							

**Many things still to understand....
over to you!**

